Mobile Call Termination and Collusion under Asymmetry *

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Abstract

This paper looks at duopolistic competition in the telecommunications industry allowing for asymmetric networks. Employing the standard Hotelling framework of horizontal product differentiation with non-linear tariffs and network based price discrimination we allow for differentiation in a second dimension. Modulo the locations, the consumers of each network operator face an asymmetry parameter that directly impacts their demands and can capture asymmetries in demand elasticities, in demand size, or even both. The implications of these asymmetries for the possibility of sustaining collusion are investigated under alternative access pricing regimes.

1 Introduction

The literature on network interconnection and pricing strategies in the telecommunications industry originating in the work of Armstrong (1998) and Laffont *et al.* (LRT 1998a,b) has generically assumed that competition takes place between symmetric networks.

This assumption of the symmetry of the two networks is a most welcome simplifying device to keep the analysis of the pricing vectors that form the Nash equilibrium of the game tractable. However, in most cases of regulatory concern it is a later entrant that competes against an incumbent with an established market share and possibly also against substantial switching costs. Thus the symmetry assumption

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of previous models, being the source of various 'profit neutrality results', see LRT (1998a) and Dessein (2003), seems to be unfortunate.

Previous research in asymmetric telecommunication environments is still quite scarce. Carter and Wright (2003) show that, given that access charges have to be chosen reciprocally (i.e. symmetrically), firms may prefer them to be set at cost if size differences are pronounced. Peitz (2005) investigates the issue of asymmetric regulation but focuses on entry and consumer surplus. Hoernig (2007) finds evidence that larger firms will tend to have a larger price differential between its on- and off-net prices, but does not explicitly model access charges. Hoernig (2010) allows for asymmetric market shares with more than two firms and also looks at mobile-to-mobile call termination, however only in a symmetric setting.

As shown in Behringer (2012), one can indeed find non-reciprocal equilibrium access charges with a positive markup on termination cost as observed in regulatory practice, by assuming that such charges are chosen non-cooperatively (as in Behringer (2009)), and that networks are potentially asymmetric. This gap in the literature has been noted as early as in Armstrong (2002: 373)) and Geoffron and Wang (2008) employ the same modelling of asymmetry to investigate the effects of calling clubs. The urgency for theoretical models to accommodate these practical concerns is emphasised in Hurkens and López (2014), who modify the consumer expectations used in LRT. Other explanations for positive markups are provided in Armstrong and Wright (2009), Jullien *et al.* (2013), and Hoernig *et al.* (2014). A collective early volume dedicated to the issue of asymmetries in mobile markets in order to increase realism as demanded in a study for the European Commission (see Tera 2009: 133) is Benzoni and Geoffron (2007). Recently López and Rey (2012) also look at asymmetric duopoly (due to switching costs) with a focus on market foreclosure.

>From the very beginning, the issue of collusion has been present in the analysis of the telecommunication industry. For an introductory overview, see Peitz *et al.* (2004) and for its empirical relevance see, for example, the case of *Autorité de la Concurrence* (2005). However almost all theoretical investigations focus on the effect of access charges on the resulting retail price components only. An exception to this is the work of Höffler (2009), who looks at collusion in the classical way in an infinitely repeated Bertrand competition setting with heterogenous consumers. Again, however, the firms are assumed to be symmetric.

One of the few recent papers that have joined the issues of asymmetric firms with the incentives to collude is Baranes and Poudou (2009). It has long been the consensus that collusion is easier to sustain among symmetric firms. Their model allows for differing price sensitivities of consumer demands (e.g. resulting from switching costs) and access charges and they find that symmetry in access regulation may actually inhibit collusion. The model employs a differentiated duopoly framework.

The present paper extends this concern with direct collusion to the telecommunications industry, employing a standard horizontal differentiation Hotelling setup with two-part tariffs and network based price discrimination. It allows for a more general form of demand that encompasses various possible origins of asymmetries. The implications of asymmetries for the possibility of directly sustaining collusion are then investigated under alternative access pricing regimes.

Today, regulators intend to reduce the level of asymmetric regulation of access charges according to a *glide path*. This is especially true in the case of fixed and mobile termination rates (see *European Commission* 2009) and for the current discussions about the implementation of 'bill and keep' regimes. Thus the relevance of extending the findings in Baranes and Poudou (2009) to the particularities of the telecommunications industry follows naturally.

Section 2 presents the model in which the competitive, the collusive, and the deviation outcome are laid out, and Section 3 the respective equilibria. Section 4 and 5 investigate how the critical discount factors are affected by potential asymmetries of the networks in demand, in costs and/or in access pricing regimes. Section 6 concludes. The proofs are relegated to the Appendix.

2 Model setup

The following model uses the setting of the network competition models of Laffont *et al.* (1998) with duopolistic competition in two-part tariffs, on-net and off-net price discrimination, and balanced calling patterns. As in Carter and Wright (1999, 2003) we allow explicitly for an exogenous asymmetry between networks. In contrast, we assume that the asymmetry is directly related to the demand for calls. This implies that the asymmetry does not simply impact the fixed utility, but affects the volume of calls. We then consider the question of collusion sustainability in the telecommunications industry by focusing on the role of this demand asymmetry.

Demand asymmetry. To model the demand asymmetry between networks, we assume the consumer demand for calls to be given by $q(p, \eta)$ where p is the unit price and η is a parameter assumed to indicate the asymmetry. This parameter can represent the elasticity of demand or measure the size of the demand (or network), or can be generalised to any other type of heterogeneity with relative importance for each network. In our duopoly setting, we write η_i for the parameter describing network i, with $i = 1,2$. Considering network i, we assume that the asymmetry parameter exceeds a given level $\eta_i \geq \eta_0$. Networks are symmetric if $\eta_i = \eta_0$ ($\forall i = 1,2$) and asymmetric otherwise, where η_0 is the symmetric benchmark level for the asymmetry parameters η_i. The variable net surplus $v(p,\eta)$ a consumer gets from consuming q unit of calls is given by

$$v(p,\eta) \equiv \int_p^\infty q(\zeta,\eta)d\zeta = u(q(p,\eta),\eta) - pq(p,\eta)$$

where $u(q,\eta)$ represents the gross utility from making q calls.

We make the following standard assumptions both on demand and indirect utility[1]: $q_1(p,\eta) < 0$ and $v_1(p,\eta) = -q(p,\eta) < 0$, $\forall$ (p,η). This implies that both quantity and indirect utility are decreasing functions of unit price. Moreover we maintain the following assumption:

Assumption. *For all (p,η), the demand and indirect utility functions either satisfy A.1:* $\text{sgn}\left(q_2(p,\eta)\right) = \text{sgn}\left(v_2(p,\eta)\right) < 0$ *or, A.2:* $\text{sgn}\left(q_2(p,\eta)\right) = \text{sgn}\left(v_2(p,\eta)\right) \geq 0$.

Assumptions *A.1* and *A.2* deserve some more comment. For a given price, the effects of asymmetries on demand and indirect utility coincide. Assumption *A.1* (resp. *A.2*) implies that if the asymmetry has a decreasing impact on demand it will also decrease the consumer indirect utility and vice versa. Assume $\eta_1 \geq \eta_2$, then Assumption *A.1* implies that both demand and indirect utility derived from network 1 are always lower than those derived from network 2. The reverse holds for Assumption *A.2*.

To illustrate A 2 consider the following examples:

Example 1 (isoelastic demand): The isoelastic demand function is given by $q\left(p,\eta\right) = A\left(\eta\right)p^{-\epsilon(\eta)}$, where $A(\eta) > 0$ is the size of the demand (or network) and $\epsilon(\eta) > 1$ is the elasticity of demand. The indirect utility is then $v\left(p,\eta\right) = \frac{A(\eta)p^{1-\epsilon(\eta)}}{\epsilon(\eta)-1}$. First assume that the asymmetry weighs fully on the demand size, i.e. $A(\eta) = \eta$ and $\epsilon(\eta) = \epsilon$. Then one can see that $q_2(p,\eta) = p^{-\epsilon} > 0$ and $v_2(p,\eta) = \frac{p^{1-\epsilon}}{\epsilon-1} > 0$. Now consider that the asymmetry weighs fully on the demand elasticity, i.e. $A(\eta) = A$ and $\epsilon(\eta) = \eta$. Then $q_2(p,\eta) = -\ln(p)q(p,\eta) < 0$ and $v_2(p,\eta) = -\frac{(1+(\eta-1)\ln(p))v(p,\eta)}{(\eta-1)} < 0$, for a given price $p > 1$.

Example 2 (linear demand): Consider the linear demand function given by $q\left(p,\eta\right) = A(\eta) - B\left(\eta\right)p$, where $A(\eta) > 0$ is the size of the demand (or network) and $B(\eta) > 0$ is a slope parameter that increases[2] the elasticity of demand for a

[1] Hereafter lower indices denote the position of the argument of the function for which the partial derivative is taken.

[2] Clearly $B\left(\eta\right)$ is not the elasticity here, it writes $\epsilon = B\left(\eta\right)p/(q(p,\eta)^2$. However, increasing $B\left(\eta\right)$ increases the demand elasticity since $d\epsilon/dB\left(\eta\right) = A\left(\eta\right)p/q(p,\eta)^2 > 0$.

given price. The indirect utility function is then $v(p,\eta) = \frac{1}{2B(\eta)} q(p,\eta)^2$. Hence if we assume that the asymmetry weighs fully on the demand size, i.e. $A(\eta) = \eta$ and $B(\eta) = B$, then one can see that $q_2(p,\eta) = 1 > 0$ and $v_2(p,\eta) = q(p,\eta)/B > 0$. If the asymmetry weighs fully on the elasticity proxy parameter B, i.e. $A(\eta) = A$ and $B(\eta) = \eta$, then both demand and indirect utility are decreasing functions of η, i.e. $q_2(p,\eta) = -p < 0$ and $v_2(p,\eta) = -\frac{1}{2\eta^2} q(p,\eta)(A + \eta p) < 0$.

Network market shares and profits. We now consider competition between two networks, $i = 1, 2$ located at the opposite ends of a Hotelling unit line, where network 1 is located at $x = 0$ and network 2 is located at $x = 1$. Consumers are assumed to be uniformly distributed on that unit line and the transportation cost is denoted by $\theta > 0$ per unit. Both networks offer a two-part tariff to consumers including the fixed fee f_i, the on-net unit price p_i, and the off-net unit price $\hat{p}_i$. Given a balanced calling pattern a consumer purchasing from network i obtains a net total surplus given by

$$w_i = \alpha_i v(p_i, \eta_i) + (1 - \alpha_i) v(\hat{p}_i, \eta_i) - f_i \tag{1}$$

where α_i denotes the market share of network i.

Given the unit transportation cost θ a consumer who is identified by his location x gets an overall utility $w_1 - \theta x$ when joining network 1, and $w_2 - \theta(1 - x)$ when joining network 2. The marginal consumer between network 1 and 2 is defined by $\hat{x} \equiv (\theta + w_1 - w_2)/2\theta$.

Each network bears a fixed cost normalised to 0 and a common marginal costs c_0 at the originating or the terminating end of each call. For each unit of off-net calls from network j to network i network j pays the termination fee a_i. Thus the per unit cost of an off-net call is $c_0 + a_i$ and the per unit cost of an on-net call is $2c_0$.

We restrict attention to market conditions for which the market is fully covered by the networks. This will be the case in particular when the networks are very similar (η_1 close to η_2) and the termination fees are cost-based (a_1 and a_2 close to c_0). To ensure this formally, we assume that the networks are moderately differentiated so that,

$$\frac{2}{3} v(2c_0, \eta_0) \geq \theta \geq \frac{2}{7} v(2c_0, \eta_0) \tag{2}$$

where η_0 is the symmetric benchmark level. The market shares of the two networks are given by $\alpha_1 = \hat{x}$ and $\alpha_2 = 1 - \hat{x}$.

Setting the two-part tariff $(f_i, p_i, \widehat{p_i})$, the profit function for network i is

$$\begin{aligned} \pi_i(p_i, \hat{p}_i; p_{-i}, \hat{p}_j) &= \alpha_i \{\alpha_i q(p_i, \eta_i)(p_i - 2c_0) + (1 - \alpha_i) q(\hat{p}_i, \eta_i)(\hat{p}_i - c_0 - a_j) \\ &\quad + (1 - \alpha_i) q(\hat{p}_j, \eta_j)(a_i - c_0) + f_i\} \end{aligned}$$

where $q(p_i, \eta_i)$ and $q(\hat{p}_i, \eta_i)$ are the number of on-net and off-net calls of network i respectively.

>From (1) we have $f_i = \alpha_i v(p_i, \eta_i) + (1 - \alpha_i)v(\hat{p}_i, \eta_i) - w_i$ and we can rewrite the profit of network i as

$$\begin{aligned} \pi_i(\mathbf{p}, \mathbf{w}) &= \alpha_i\{\hat{x}q(p_i, \eta_i)(p_i - 2c_0) + (1 - \alpha_i)q(\hat{p}_i, \eta_i)(\hat{p}_i - c_0 - a_j) + \\ &+(1 - \alpha_i)q(\hat{p}_j, \eta_j)(a_i - c_0) + \alpha_i v(p_i, \eta_i) + (1 - \alpha_i)v(\hat{p}_i, \eta_i) - w_i\} \end{aligned} \tag{3}$$

We next define two shorthand notations for the indirect utility difference functions $V(y, z, i, j) \equiv v(y, \eta_i) - v(z, \eta_j)$ and revenues $R(y, z, t, i) \equiv (y - z)q(t, \eta_i)$.

Collusion. As is standard in the classical analysis of tacit collusion (Friedman 1971), we consider an infinitely repeated tariff competition game. The punishment strategy for a given operator corresponds to a trigger strategy with reversion to the static competitive equilibrium. We denote the individual profit gained from a punishment strategy (Nash reversion to competition) as π_i^* and the individual collusion profit as π_i^C. Finally the individual profit gained from deviating from the collusive agreement is π_i^D. As is well known, the fully collusive outcome can be sustained as a subgame-perfect equilibrium of the infinitely repeated game if the intertemporal discount factor δ is sufficiently large, i.e.

$$\delta_i \geq \widehat{\delta} = \max\{\widehat{\delta}_1, \widehat{\delta}_2\} \tag{4}$$

where $\widehat{\delta}$ denotes the critical discount factor and $\widehat{\delta}_i = (\pi_i^D - \pi_i^C)/(\pi_i^D - \pi_i^*)$ represents the critical discount factor for network i.

We next investigate the levels and variations of the critical discount factor $\widehat{\delta}$ with respect to both the reciprocity in the access charge regulation and the potential asymmetry of the networks. This will help us to asses how incentives to collude are driven by these two features in this industry. Note that if, for whatever reason, the critical discount factor decreases, firms are able to collude for a larger range of individual discount factors and conversely. As a result any factor that pushes down the critical discount factor should be considered as a factor that *facilitates collusion* in the industry. If the critical discount factor is pushed up, the factor *inhibits collusion.* Our aim is to identify how asymmetric access charge regulation is such a facilitating or inhibiting factor when networks become more asymmetric. To perform this analysis we will next look at the equilibrium outcomes for each operator corresponding to the three different market configurations.

3 Equilibrium outcomes

In this section we determine the equilibrium outcomes for each market configuration (competition, collusion, and deviation). Let us start with the competitive outcomes.

The competitive outcomes. This situation is the one studied by Laffont *et al.* (1998b). The equilibrium fixed fee and the price vector components of network i satisfy $(p_i^*, \hat{p}_i^*, f_i) = \arg\max_{p_i, \hat{p}_i, f_i} \pi_i (p_i, \hat{p}_i; p_j, \hat{p}_j)$. The result of the maximization is stated in the following Lemma.

Lemma 1 (LRT 1998a.)**.** *The equilibrium unit prices and the fixed fee of network i in the competitive setting are:*

(i) $p_i^* = 2c_0$ *and* $\hat{p}_i^* = c_0 + a_j$

(ii) $f_i = \dfrac{\pi_i^*}{\alpha_i^*} - (1-\alpha_i^*) R\left(a_i, c_0, \hat{p}_j^*, -i\right).$

Lemma 1 states the standard results for the competitive equilibrium prices. Equilibrium unit prices are equal to their respective marginal costs. Hence on-net prices are set at the total marginal cost of an on-net call $(2c_0)$ and off-net prices are set to their marginal cost including the unit termination fee of the competing network $(c_0 + a_j)$, the total 'perceived marginal cost'. Equilibrium fixed fees are then used by networks to extract surplus from consumers.

It follows that equilibrium market shares as determined by the marginal consumer are

$$\alpha_1^* = \frac{\theta + V\left(\widehat{p}_1^*, p^*, 1, 2\right) - f_1 + f_2}{2\theta + V\left(\widehat{p}_1^*, p^*, 1, 1\right) + V\left(\widehat{p}_2^*, p^*, 2, 2\right)} \quad \text{and } \alpha_2^* = 1 - \alpha_1^*$$

Using (3), we obtain the competitive equilibrium profit of network i,

$$\begin{aligned} \pi_i^*\left(a_i, a_j, \eta_i, \eta_j\right) &= \frac{2\theta + \Sigma_{k=1}^{k=2} V\left(\widehat{p}_i^*, p^*, k, k\right) + R\left(a_i, c_0, \hat{p}_j^*, -i\right)}{\left(6\theta + 2\Sigma_{k=1}^{k=2} R\left(a_{-k}, c_0, \hat{p}_k^*, k\right) + 3\Sigma_{k=1}^{k=2} V\left(\widehat{p}_k^*, p^*, k, k\right)\right)^2} \\ &\quad \times \left(3\theta + 2V\left(\widehat{p}_i^*, p^*, i, -i\right) + V\left(\widehat{p}_{-i}^*, p^*, -i, i\right) + \Sigma_{k=1}^{k=2} R\left(a_{-k}, c_0, \hat{p}_k^*, k\right)\right)^2 \end{aligned}$$

We highlight the fact that these equilibrium profits are functions of the termination charges (a_1, a_2) and the elasticity parameters (η_1, η_2). Note that when termination charges are cost based and symmetry holds, the profit of network i is simply equal to $\pi_i^*\left(c_0, c_0, \eta_0, \eta_0\right) = \theta/2$.

The collusive outcomes. In order to determine the fully collusive outcome we assume that the price vector maximises the joint profit subject to a participation constraint for all consumers. Then collusive unit prices and the fixed fee result from the constrained maximization problem $\max_{\mathbf{p},\mathbf{f}} \pi_1(\mathbf{p},\mathbf{w}) + \pi_2(\mathbf{p},\mathbf{w})$ s.t. $U_k(x) \geq 0$. The solution is as follows.

Lemma 2. *The equilibrium unit prices and the fixed fee of network i in the collusive setting are*

(i) $p_i^C = \hat{p}_i^C = 2c_0$, *for* $i = 1, 2$,

(ii) $f_i^C = \frac{1}{4}\left(3v(2c_0, \eta_i) + v(2c_0, \eta_j)\right) - \frac{1}{2}\theta$.

Note that this collusive equilibrium corresponds to the multiproduct monopolistic outcome when a two-part tariff is charged. All collusive marginal prices are set to the marginal cost in order to enhance the network's productive efficiency and the fixed fees are used to capture almost the entire consumer's surplus (and the entire surplus of the indifferent consumer).

Using (3) and substituting in the equilibrium collusive prices, we obtain the equilibrium profit of network i in the collusive setting as:

$$\pi_i^C\left(a_i, a_{-i}, \eta_i, \eta_{-i}\right) = \frac{\left(2\theta + V\left(p^C, p^C, i, -i\right)\right)\left(2\theta + V\left(p^C, p^C, -i, i\right)\right)}{16\theta^2}$$
$$\times\left(R\left(a_i, c_0, p^C, -i\right) - R\left(a_{-i}, c_0, p^C, i\right) + \frac{\theta\left(3v\left(p^C, \eta_i\right) + v\left(p^C, \eta_{-i}\right) - 2\theta\right)}{\left(2\theta + V\left(p^C, p^C, -i, i\right)\right)}\right)$$

When the termination fees are cost based and symmetry holds, the profits are simply $\pi_i^C\left(c_0, c_0, \eta_0, \eta_0\right) = \left(2v\left(2c_0, \eta_0\right) - \theta\right)/4$, which is positive under (2).

The deviation outcomes. We assume without loss of generality that it is network i that deviates from the collusive outcome. The deviation unit prices and the fixed fee are then derived from the constrained maximization problem $\max_{p_i, \hat{p}_i, f_i} \pi_i\left(p_i, \hat{p}_i, p_{-i}^C, \hat{p}_{-i}^C, f_i, f_{-i}^C\right)$ s.t. $U_k(x) \geq 0$. The solution is as follows.

Lemma 3. *The equilibrium unit prices and the fixed fee in the deviation setting are:*

(i) $p_i^* = 2c_0$ *and* $\hat{p}_i^* = c_0 + a_j$

(ii) $f_i^D = \dfrac{\pi_i^D}{\alpha_i^D} - (1 - \alpha_i^D) R\left(a_i, c_0, p^*, -i\right)$.

Note that network i deviates from the collusive equilibrium using its fixed fee while leaving unit on-net and off-net prices unchanged. In doing so network i can attract more consumers and increase its overall profit.

Using (3), we find that the equilibrium deviation profit of network i is

$$\pi_i^D\left(a_i, a_{-i}, \eta_i, \eta_{-i}\right) = \frac{1}{64}\,\frac{\left(4\,v\left(\hat{p}_i^*, \eta_i\right) + 4R\left(a_i, c_0, \hat{p}_{-i}^*, -i\right) + V(p^*, p^*, i, -i) + 2\,\theta\right)^2}{R\left(a_i, c_0, \hat{p}_{-i}^*, -i\right) + V(\hat{p}_i^*, p^*, i, i) + 2\,\theta}$$

Again with reciprocal termination fees, deviation profits are equal for the two operators. For both of them, cost-based termination fees and symmetry imply profits of $\pi_i^D(c_0, c_0, \eta_0, \eta_0) = (2v(2c_0, \eta_0) + \theta)^2 / 32\theta$.

A thorny issue when looking at deviation outcomes is that monopolization can occur ex-post, with the deviating firm remaining the only firm in the market. To avoid this, we restrict our model to market conditions that preserve a duopolistic structure when firms deviate. With condition (2), network i's market share α_i^D always belongs to the interval $[0,1]$ when $(a_i, a_{-i}, \eta_i, \eta_{-i}) = (c_0, c_0, \eta_0, \eta_0)\ \forall\ i$.

We are now in a position to construct and study the critical discount factor as defined in (4). However, due to the tedious expressions for the profits we will not provide a complete exposition and characterization of this threshold. Note that the individual thresholds $\widehat{\delta}_i(a_i, a_{-i}, \eta_i, \eta_{-i})$ defined in (4) are implicit functions of $(a_1, a_2, \eta_1, \eta_2)$, as are the profits.

Equipped with this framework, we now proceed to the analysis of the sustainability of price collusion. We focus on the effect of the asymmetry parameter (influencing the demand elasticity or the network size) on the incentives for operators to collude. In particular, we examine how asymmetric regulation of the termination fee affects the sustainability of collusion. However a complete analysis for all values of access charges (a_1, a_2) and asymmetry parameters (η_1, η_2) involves strong nonlinearities that make the analysis very tedious. Hence, as is standard in the literature, we will analyze asymmetric regulation locally around cost-based termination fees. That is, we will study how a departure from cost-based regulation will affect the sustainability of collusion, depending on the potential asymmetry of the networks and the regulation, allowing for reciprocal or asymmetric termination fees.

The table 1 summarises the analysis we conduct hereafter, and it emphasizes that instead of asymmetric regulation, we may as well think of an asymmetric (marginal) cost difference between the networks under symmetric regulation. More precisely, that not only do we model asymmetric regulation under symmetric cost or symmetric regulation under asymmetric cost but also (because it is only the net magnitude that matters) the full continuum of cases in-between. This is of importance as it is well known (see for example Ivaldi *et al.* (2007)) that low-cost firms in the industry have less incentives to collude and that cost asymmetry inhibits tacit collusion. This leads to the conventional doctrine that ‘it is easier to collude among equals’ when costs are considered.

To better isolate the pure effect of network asymmetry, we start assuming that the networks are symmetric, so that $\eta_1 = \eta_2 = \eta_0$, and study the impact of differ-

		Cost Asymmetry	
		No	Yes
Network	No	Benchmark	$\eta_1 = \eta_2$ and $a_1 > a_2$
Asymmetry	Yes	$\eta_1 > \eta_2$ and $a_1 = a_2$	$\eta_1 > \eta_2$ and $a_1 > a_2$

Table 1: Dual asymmetric framework.

ent regulatory termination fee regimes (reciprocal *v.s.* asymmetric regulation) on collusion.

4 Symmetric networks

This section analyzes the effects of asymmetric regulation[3], $a_1 \geq a_2 = c_0$, for the case that networks are symmetric, $\eta_1 = \eta_2 = \eta_0$. We first examine the reciprocal regulation regime and then asymmetric regulation.

4.1 Reciprocal regulation as a benchmark

Considering reciprocal regulation, $a_1 = a_2 = a$, with symmetric networks and a cost-based termination fee $a = c_0$, it can be shown from (4) that the critical discount factor becomes

$$\hat{\delta}\left(c_0, c_0, \eta_0, \eta_0\right) = \frac{2v(2c_0, \eta_0) - 3\theta}{2v(2c_0, \eta_0) + 5\theta}$$

Note that this corresponds to the long run situation in which the initial advantages of the incumbent (which may result from brand recognition or switching costs) are overcome and the networks' termination fees are regulated to follow, for example, the so-called 'glide path' to cost envisaged by the European Commission.

The critical discount factor $\widehat{\delta}$ is then decreasing with the transportation cost θ, which plays the role of a network differentiation parameter. If θ is larger, goods become less substitutable for consumers, i.e. product differentiation is higher, which implies that it is easier to sustain collusion. Why is this the case? Omitting arguments the competitive profit is

$$\pi_i^* = \frac{\theta}{2}$$

and, as usual in the Hotelling model, it is strongly increasing in θ. Note also that for $\theta > \frac{2}{3}v(2c_0, \eta)$, there is no incentive to deviate as

$$\pi_i^D = \frac{\left(2v(2c_0, \eta_0) + \theta\right)^2}{32\theta} < \pi_i^* = \frac{\theta}{2}.$$

[3]Since networks are assumed to be symmetric we could either consider the case $a_1 \geq a_2 = c_0$ or $a_2 \geq a_1 = c_0$.

Hence, we again need (2) to hold[4]. The collusive profit is

$$\pi_i^C = \frac{2v\left(2c_0, \eta_0\right) - \theta}{4}$$

Note that the collusive profit is decreasing in θ, i.e. a higher degree of product differentiation reduces the total profit of an already colluding cartel, whereas the effect on the deviation profit is ambiguous. However, the differences in the numerator $\pi_i^D - \pi_i^C$ and the denominator $\pi_i^D - \pi_i^*$ are decreasing in θ and, from the overall result, we know that the effect of the numerator dominates making deviation from the collusive agreement less attractive.

Considering now reciprocal regulation, $a_1 = a_2 = a$, we have the following result.

Proposition 1. *In a symmetric network setting and with cost-based regulation, the critical discount factor $\widehat{\delta}$ is decreasing in the reciprocal access charge.*

We therefore find that increasing a reciprocal termination fee locally around cost facilitates collusion. Conversely, reducing reciprocal termination fee to cost, as under the European 'glide path' will make collusion harder to sustain. This first result underlines the collusive effect of reciprocal regulation in an infinitely repeated tariff competition game and confirms results of the standard literature on competition between interconnected networks stated by LRT (1998a) and Armstrong (1998). Indeed, when the access charge increases from its cost-base, off-net prices reach a higher level for both operators due to reciprocal access charges. Then operators compete through fixed fees which reduces their competitive profits. As a consequence, they have a higher incentive to collude. This proposition shows that following the European 'glide path' yields a double-dividend for the society when networks are (or have become) symmetric.

4.2 Asymmetric regulation

We now consider asymmetric regulation, i.e. non-reciprocal termination fees $a_1 \geq c_0$ and $a_2 = c_0$. Operators then do not have the same incentives to collude and their critical discount factors take different values: $\widehat{\delta}_1 \neq \widehat{\delta}_2$, even though networks are fully symmetric. Of course such termination fees will have an impact on the incentive to collude for both networks and thus on the critical discount factor $\hat{\delta}$. Assume $a_2 = c_0$. The following proposition states the result for a slight deviation of network 1's termination fee from its cost-based level. Define $\widetilde{\theta} \equiv \frac{6}{13} v(2c_0, \eta_0)$. Then we find that

[4] For $\theta = \frac{2}{3} v(2c_0, \eta_0)$ we have $\pi_i^D = \frac{1}{3} v(2c_0, \eta_0)$ and $\pi_i^C = \frac{1}{3} v(2c_0, \eta_0)$ but also $\pi_i^* = \frac{1}{3} v(2c_0, \eta_0)$ then $\delta_i^* = 0$ and one can sustain collusion for any discount factor.

Proposition 2. *With symmetric networks, assuming that cost-based termination fees are regulated asymmetrically, there exists a threshold* $\widetilde{\theta}$ *such that* $\frac{\partial \hat{\delta}(c_0,c_0,\eta_0,\eta_0)}{\partial a_1} \leq 0$ *if* $\theta \geq \widetilde{\theta}$ *and* $\frac{\partial \hat{\delta}(c_0,c_0,\eta_0,\eta_0)}{\partial a_1} > 0$ *otherwise.*

When networks are symmetric, asymmetric regulation will facilitate collusion whenever product differentiation is sufficiently high. Conversely, reducing asymmetric regulation towards a 'glide path' regime will make collusion harder to sustain. However, this is no longer the case when product differentiation is low. Then competition in fixed charges is fierce, the effect of asymmetric regulation is diluted, and an increasing wedge between on-net and off-net access margins will improve competitive and deviation profits relative to collusive ones. Thus an increase in the access charge will make collusion harder to sustain.

5 Asymmetric networks

We now allow for asymmetric networks with differing asymmetry parameters. Suppose that network 1 now has a higher value of the asymmetry parameter and network 2 still has the lower (previously symmetric) value η_0, so that $\eta_1 > \eta_2 = \eta_0$. Different cases may arise depending on whether network 1 benefits from asymmetric regulation or not. As in the previous section we assume that asymmetric regulation benefits network 1 so $a_1 > a_2 = c_0$. Following A 2, illustrated by *Examples* 1 and 2 the asymmetry between networks can represent two kinds of situations. Firstly the network asymmetry may fall fully on the demand elasticity (Assumption $A.1$). In this case asymmetric regulation benefits the network with the higher elasticity. It has often been taken for granted that new entrants in the mobile market face a higher elasticity than incumbents because of switching costs or first mover advantages. Asymmetric regulation can then be considered as a way to reduce the competitive disadvantage of the high elasticity network (i.e. new entrant) by offering the possibility of charging a higher termination fee than the incumbent. This has been allowed for in the European regulation of mobile termination rates. Network asymmetry may however also fall on the demand or network size (Assumption $A.2$). This second case then represents a situation in which asymmetric regulation benefits the operator with the larger demand or network. Such a kind of asymmetric regulation never happened in the termination fee regulation policy. This is probably because asymmetric regulation has always been implemented to limit the advantage of the incumbent and favour competition of new entrants[5]. However asymmetric

[5]Indeed one could imagine that when $A.2.$ holds, the empirically more plausible case would be that the smaller network is allowed a higher access charge (i.e. $a_2 > a_1$). However, this constellation then creates the same 'mirror empirical' problem if $A.1.$ holds.

regulation does have an effect on the sustainability of collusion when favoring the incumbent as asymmetric regulation can reduce the operators' incentives to stick to a collusive agreement. In the following, we investigate how the critical discount factor is affected by both network asymmetry and different regulatory regimes.

5.1 Reciprocal regulation

Assume first that reciprocal regulation applies. Termination fees are then cost-based and $a_1 = a_2 = c_0$. The critical discount factors for network i follows.

$$\hat{\delta}_i\left(c_0, c_0, \eta_i, \eta_{-i}\right) = \frac{9\left(v(2c_0, \eta_i) + 3v(2c_0, \eta_{-i}) - 6\theta\right)^2}{\left(23v(2c_0, \eta_i) - 11v(2c_0, \eta_{-i}) + 30\theta\right)\left(7v(2c_0, \eta_i) + 5v(2c_0, \eta_{-i}) - 18\theta\right)} \tag{5}$$

The next result compares the incentive for collusion of both networks and presents the critical discount factor $\widehat{\delta}$.

Lemma 4. *Assuming a small asymmetry between networks (η_1 is in a right neighbourhood of $\eta_2 = \eta_0$):*

(i) if A.1 holds, then $\hat{\delta}_1 > \hat{\delta}_2$

(ii) if A.2 holds, then $\hat{\delta}_1 < \hat{\delta}_2$.

When networks are asymmetric and the reciprocal termination fee is cost-based, the critical discount factor is that belonging to the operator who, because of a (perceived) differentiation in networks or the installed user base, is structurally able to lower the consumer's surplus at each price. This finding implies that the more disadvantaged firm is more likely to break a collusive agreement, i.e. its relative optimal deviation profits are higher. This is in line with the common precept that collusion is easier to sustain among equals.

Let us now assume reciprocal termination fees, $a_1 = a_2 = a \geq c_0$, and a small termination fee mark-up. In Proposition 3 after, we focus on the interplay between network asymmetries and reciprocal termination fees.

Proposition 3. *(i) When A.1. holds, there exists a level $\underline{v}$ of $v_2(p,\eta)$ and two values $\theta_1 = \underline{x}_1 v(2c_0, \eta_0)$ and $\theta_2 = \underline{x}_2 v(2c_0, \eta_0)$ where $\underline{x}_2 > \underline{x}_1 > 6/13$, such that:*

(i.a) if $\underline{v} < v_2(p,\eta) < 0$ and $\theta \in [\theta_1, \theta_2]$ then

$$\frac{\partial^2 \hat{\delta}_1\left(c_0, c_0, \eta_0, \eta_0\right)}{\partial a \partial \eta_1} < 0$$

(i.b) if $\underline{v} < v_2(p,\eta) < 0$ and $\theta \notin [\theta_1, \theta_2]$ or if $v_2(p,\eta) \leq \underline{v}$ $\forall\ \theta$ then

$$\frac{\partial^2 \hat{\delta}_1\left(c_0, c_0, \eta_0, \eta_0\right)}{\partial a \partial \eta_1} > 0.$$

(ii) When A.2. holds then unambiguously

$$\frac{\partial^2 \hat{\delta}_2 (c_0, c_0, \eta_0, \eta_0)}{\partial a \partial \eta_1} < 0.$$

As shown in Proposition 1 with homogeneous networks, reciprocal access charges above cost have a facilitating effect for collusion in the industry. Proposition 3 shows that this facilitating effect is not always enhanced further by network asymmetries. It depends on both the level of product differentiation θ and the impact of network asymmetries on the surplus $v(p, \eta)$. It is worth pointing out that depending on those fundamentals, the critical discount factor can be reduced when the reciprocal access charge is slightly raised above cost: more asymmetries do not systematically inhibit collusion, but this will be the case when asymmetries markedly deteriorate surpluses, i.e. $v_2(p, \eta) \leq \underline{v} < 0$.

For the case that the asymmetry impacts mostly on network scale, we get a clear cut result. We already know that it is the smaller firm that has the stronger incentives to deviate. We now see that a further increase in this scale asymmetry will amplify the negative impact of an increasing wedge on competitive profits, thus increasing the severity of punishment, making deviation less attractive.

5.2 Asymmetric regulation

Assuming non-reciprocal charges $a_1 \geq c_0$ and $a_2 = c_0$, from Proposition 2 we know the effects of access margins on collusion (around cost-based pricing) and from Proposition 3 we have results on the effects of network asymmetries. Using these findings we now investigate how the possibilities of sustaining collusion are affected by slight demand asymmetries.

Proposition 4. *When A.1 holds, there exists a level of θ i.e. $\hat{\theta} < \frac{2}{7} v(2c_0, \eta_0)$ such that*

$$\frac{\partial^2 \hat{\delta}_1 (c_0, c_0, \eta_0, \eta_0)}{\partial a_1 \partial \eta_1} \lesseqgtr 0 \textit{ if } \theta \gtreqless \hat{\theta}$$

When A.2 holds, then unambiguously

$$\frac{\partial^2 \hat{\delta}_2 (c_0, c_0, \eta_0, \eta_0)}{\partial a_1 \partial \eta_1} < 0.$$

When *A.1.* holds, *i.e.* $v_2(p, \eta) < 0$, a slight network asymmetry in the sense that $\eta_1 > \eta_0$ strengthens the facilitating effect of a positive off-net access margin on collusion if $\theta \geq \hat{\theta}$, but weakens this effect otherwise. When *A.2.* holds, *i.e.* $v_2(p, \eta) > 0$, then a similar slight network asymmetry always strengthens the facilitating effect of positive off-net access margins on collusion. Hence, in both cases, if product differentiation is high enough, more asymmetries do not inhibit collusion. Thus, the case where the asymmetry impacts mostly on the network scale is analogous to the case with symmetric regulation.

6 Conclusion

For a differentiated Bertrand duopoly setting, Baranes and Poudou (2009) show that cost symmetry may inhibit collusion, so that the common precept that it is easier to collude amongst equals does not always hold. In this paper, we looked at a differentiated Hotelling duopoly model of the kind used by LRT (1998a,b) for the telecommunications industry with a potential asymmetry from differences in demand elasticities and/or installed bases that may result from differences in firm histories.

We found that with homogenous networks, i.e. the long-run competitive outcome in this industry which will eventually arise long enough after the technological breakthroughs starting from the beginning of this century, and with a cost-based access charge regime, a larger reciprocal on-net off-net margin will actually improve the possibilities for collusion. We also found that reducing reciprocal access charges to true cost, as aimed at by the European 'glide path' envisaged by the Commission - and Ofcom in the UK, see (2010) - will make collusion harder to sustain for homogenous networks. Hence this policy can be seen to yield a double dividend.

In a competitive setting with heterogenous networks, i.e. what can be seen as the medium term outcome, where competition will have fostered and regulation can therefore be relaxed, a higher degree of differentiation in demand elasticities actually improves firms' profits and it is the firm facing the larger demand elasticity (usually the incumbent) that is more likely to have a level of impatience that leads to the breach of a collusive agreement. This has implications for medium term policy, as measures aimed at equalizing (consumers' perceptions of the) networks may actually improve firms' possibilities for collusion. The finding is thus in line with the common precept that it is easier to sustain collusion amongst equals, and should keep regulators on their toes.

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Appendix

• **Proof of Lemma 1.** Given in LRT (1998a).■

• **Proof of Lemma 2.** Using (3), one can form the joint profit $\pi_1(\mathbf{p},\mathbf{w})+\pi_2(\mathbf{p},\mathbf{w})$ and show that *(ii)* is independent of (a_1,a_2), so that the relevant first order conditions

$$\frac{\partial(\pi_1(\mathbf{p},\mathbf{w})+\pi_2(\mathbf{p},\mathbf{w}))}{\partial p_i}=0, \frac{\partial(\pi_1(\mathbf{p},\mathbf{w})+\pi_2(\mathbf{p},\mathbf{w}))}{\partial \hat{p}_i}=0 \ \forall i\in\{1,2\}$$

imply $p_1^C=\hat{p}_1^C=2c_0$. Then from $\hat{x}=(\theta+w_1-w_2)/2\theta$ we have that

$$\hat{x}=\alpha=\frac{1}{2}+\frac{w_1-w_2}{2\theta}$$

and with (1) using $\mathbf{p}^C$ we can calculate

$$\hat{\alpha}^C=\frac{1}{2}+\frac{1}{2\theta}(v(2c_0,\eta_1)-f_1-v(2c_0,\eta_2)+f_2)$$

Setting the utility of the marginal consumer to zero,

$$\hat{U}=\hat{\alpha}^C v(2c_0,\eta_1)+(1-\hat{\alpha}^C)v(2c_0,\eta_2)-f_1=0$$

we can determine the collusive fixed charge:

$$f_1^C=v(2c_0,\eta_1)+v(2c_0,\eta_2)-f_2-\theta$$

Putting this into $\Pi=\pi_1(\mathbf{2c}_0,\mathbf{w})+\pi_2(\mathbf{2c}_0,\mathbf{w})$ and maximizing it w.r.t. f_2, s.t. $U_2(\alpha_1^C)\geq 0$, yields

$$\mathbf{f}^C=\begin{pmatrix} f_1^C \\ f_2^C \end{pmatrix}=\frac{1}{4}\begin{pmatrix} 3v(2c_0,\eta_1)+v(2c_0,\eta_2)-2\theta \\ 3v(2c_0,\eta_2)+v(2c_0,\eta_1)-2\theta \end{pmatrix}. \blacksquare$$

• **Proof of Lemma 3**. W.l.o.g. assume that $i = 1$. A similar proof holds if $i = 2$. From (3) the relevant first order conditions

$$\frac{\partial(\pi_1\left(p_1, \hat{p}_1, p_2^C, \hat{p}_2^C, f_1, f_2^C\right))}{\partial p_1} = 0 \text{ and } \frac{\partial(\pi_1\left(p_1, \hat{p}_1, p_2^C, \hat{p}_2^C, f_1, f_2^C\right))}{\partial \hat{p}_1} = 0$$

imply

$$\left(p_1^D, \hat{p}_1^D\right) = (p_1^*, \hat{p}_1^*) = (2c_0, c_0 + a_2)$$

i.e. optimal deviation yields 'perceived marginal cost' pricing just as in monopoly. The deviant profit given $\left(p_1^D, \hat{p}_1^D\right)$ is

$$\pi_1^D = \hat{\alpha}^D\left((1-\hat{\alpha}^D)\hat{q}_2(a_1 - c_0) + f_1^D\right)$$

and thus

$$f_1^D = \frac{\pi_1^D}{\alpha_1^D} - (1-\alpha_1^D)q(\hat{p}_2^C, \eta_2)(a_1 - c_0)$$

with $\hat{p}_2^C = 2c_0 = p^*$. Moreover as $\hat{x} = \alpha$ with (1) and using $\left(p_1^D, \hat{p}_1^D\right)$, we can calculate

$$\hat{\alpha}^D = \frac{\theta - f_1 + f_2 - v(2c_0, \eta_2) + v(c_0 + a_2, \eta_1)}{2\theta - v(2c_0, \eta_1) + v(c_0 + a_2, \eta_1)}$$

Setting the utility of the marginal consumer to zero

$$\hat{U} = \hat{\alpha}^D v(2c_0, \eta_1) + (1-\hat{\alpha}^D)v(c_0 + a_2, \eta_2) - f_1 - \theta\hat{\alpha}^D = 0$$

this can be solved for

$$f_2 = \frac{\theta f_1 + (\theta - v(2c_0, \eta_1) - v(2c_0, \eta_2)\theta - v(c_0 + a_2, \eta_1)v(2c_0, \eta_2) + v(2c_0, \eta_1)v(2c_0, \eta_2)}{v(2c_0, \eta_1) - v(c_0 + a_2, \eta_1) - \theta}.$$

Plugging this into the deviant profit

$$\pi_1^D = \hat{\alpha}_1^D\left((1-\hat{\alpha}_1^D)\hat{q}_2(a_1 - c_0) + f_1^D\right)$$

and maximizing over f_1, one finds the optimal deviation profit,

$$\pi_1^D = \frac{1}{64}\frac{\left(4\,v\left(\hat{p}_1^*, \eta_1\right) + v(p^*, \eta_1) - v(p^*, \eta_2) + 4\left(a_1 - c_0\right)q\left(\hat{p}_2^*, \eta_2\right) + 2\,\theta\right)^2}{v\left(\hat{p}_1^*, \eta_1\right) - v(p^*, \eta_1) + (a_1 - c_0)q\left(\hat{p}_2^*, \eta_2\right) + 2\,\theta}.$$

Note that when firms are homogeneous and access prices are cost-based,

$$\hat{\alpha}_1^D = \frac{2v(2c_0, \eta_0) + \theta}{8} = 1 - \hat{\alpha}_2^D.$$

It can be straightforwardly checked that $\hat{\alpha}_i^D \in [0, 1]$ if (2) holds.

• **Proposition 1:** Using $\hat{\delta}_i = (\pi_i^D - \pi_i^C)/(\pi_i^D - \pi_i^*)$ with the profit terms for homogenous firms and reciprocal non-cost based access charges we take the derivative with respect to a and replace the access charge with the true cost term c_0, to find

$$\frac{\partial \hat{\delta}\left(c_0, c_0, \eta_0, \eta_0\right)}{\partial a} = -8\theta \frac{q(2c_0, \eta_0)}{(2v(2c_0, \eta_0) + 5\theta)^2} < 0.$$

■

• **Proof of Proposition 2.** Denote the difference between both individual critical discount factors by $\Delta\left(a_1, a_2, \eta_1, \eta_2\right) \equiv \hat{\delta}_1\left(a_1, a_2, \eta_1, \eta_2\right) - \hat{\delta}_2\left(a_2, a_1, \eta_2, \eta_1\right)$. Around the point of cost based access pricing for a_2, the variation of the difference between the critical discount factor $\hat{\delta}_1 - \hat{\delta}_2$ w.r.t. a_1 evaluated for $a_1 = c_0$ is

$$\lim_{a_1 \to c_0} \frac{\partial \Delta\left(a_1, c_0, \eta_0, \eta_0\right)}{\partial a_1} = -\frac{16}{3} \frac{q(2c_0, \eta_0)(3v(2c_0, \eta_0) - 5\theta)}{(2v(2c_0, \eta_0) + 5\theta)^2}.$$

It is positive if $\theta \geq \frac{3}{5}v(2c_0, \eta_0)$ but negative if $\theta < \frac{3}{5}v(2c_0, \eta_0)$. Hence if $\theta \geq \frac{3}{5}v(2c_0, \eta_0)$, then δ_1^* is the relevant critical discount factor as $\hat{\delta}_1 = \max\{\hat{\delta}_1, \hat{\delta}_2\}$. Taking the derivative with respect to a_1 for $a_1 = c_0$ leads to

$$\frac{\partial \hat{\delta}_1\left(c_0, c_0, \eta_0, \eta_0\right)}{\partial a_1} = -\frac{4}{3} \frac{q(2c_0, \eta_0)(6v(2c_0, \eta_0) - 7\theta)}{(2v(2c_0, \eta_0) + 5\theta)^2} < 0 \;\forall \theta.$$

For $\theta < \frac{3}{5}v(2c_0, \eta_0)$ the relevant critical discount factor is then δ_2^* and the derivative with respect to a_1 for $a_1 = c_0$ is

$$\frac{\partial \hat{\delta}_2\left(c_0, c_0, \eta_0, \eta_0\right)}{\partial a_1} = \frac{4}{3} \frac{q(2c_0, \eta_0)(6v(2c_0, \eta_0) - 13\theta)}{(2v(2c_0, \eta_0) + 5\theta)^2} \lesseqgtr 0 \text{ if } \theta \gtreqless \tilde{\theta}.$$

where $\tilde{\theta} = \frac{6}{13}v(2c_0, \eta_0)$. ■

• **Proof of Lemma 4.** Let us consider $\left(a_1, a_2, \eta_1, \eta_2\right) = \left(c_0, c_0, \eta_1, \eta_0\right)$. From (5),

$$\begin{aligned}
\hat{\delta}_1\left(c_0, c_0, \eta_1, \eta_0\right) &= 9 \frac{\left(3\nu^0 + \nu^1 - 6\,\theta\right)^2}{\left(7\nu^1 + 5\nu^0 - 18\,\theta\right)\left(23\nu^1 - 11\nu^0 + 30\theta\right)} \\
\hat{\delta}_2\left(c_0, c_0, \eta_0, \eta_1\right) &= 9 \frac{\left(3\nu^1 + \nu^0 - 6\,\theta\right)^2}{\left(7\nu^1 + 5\nu^0 - 18\,\theta\right)\left(23\nu^1 - 11\nu^0 + 30\theta\right)}
\end{aligned}$$

can be derived where $\nu^0 = v(2c_0, \eta_0)$ and $\nu^1 = v(2c_0, \eta_1)$. Thus we can derive $\Delta\left(c_0, c_0, \eta_1, \eta_0\right)$ and consider a slight increase in η_1 above η_0.Then we have

$$\frac{\partial \Delta\left(c_0, c_0, \eta_0, \eta_0\right)}{\partial \eta_1} = -\frac{16}{3} v_2(2c_0, \eta_0) \frac{3\nu^0 - \theta}{\left(2\nu^0 + 5\theta\right)^2}. \tag{A.1}$$

As θ has been assumed to take values below $\frac{2}{3}\nu^0$, the sign of this derivative is exactly the opposite of the sign of $v_2(p, \eta)$. Hence it depends on assumptions *A.1* and *A.2*. ■

• **Proof of Proposition 3.** If *A.1* holds, i.e. $v_2(p,\eta) < 0$ and $q_2(p,\eta) < 0$, the following second order cross-partial derivative tells us how this reciprocal access pricing effect is modified by an increase in network asymmetry, that is

$$\frac{\partial^2 \hat{\delta}_1 \left(c_0, c_0, \eta_0, \eta_0\right)}{\partial a \partial \eta_1} = \frac{4}{3} \frac{\left(6\nu^0 - 13\theta\right)}{\left(2\nu^0 + 5\theta\right)^2} q_2 - \frac{8}{9} \frac{\left(8(\nu^0)^2 - 154\theta\nu^0 + 117\theta^2\right) q^0}{\left(2\nu^0 - 3\theta\right)\left(2\nu^0 + 5\theta\right)^3} v_2$$

with $\nu^0 = v(2c_0, \eta_0)$, $q^0 = q(2c_0, \eta_0)$, $v_2 = v_2(2c_0, \eta_0)$ and $q_2 = q_2(2c_0, \eta_0)$. Letting $\theta = xv^0$ for $x \leq \frac{2}{3}$, one first can see easily see that $\left(8 - 154x + 117x^2\right)\left(v^0\right)^2 < 0$ if (2) holds, i.e. if $x \in [\frac{2}{7}, \frac{2}{3}]$. Hence if $x \in [\frac{2}{7}, \frac{6}{13}]$ then unambiguously (omitting arguments) $\partial^2 \delta_1^* / \partial a \partial \eta_1 < 0$. However if $x > \frac{6}{13}$, we can find values $\underline{y}\left(x\right)$ of v_2 such that $\partial^2 \delta_1^* / \partial a \partial \eta_1$ reaches zero for each $x \in]\frac{6}{13}, \frac{2}{3}]$. This leads to solving $\partial^2 \delta_1^* / \partial a \partial \eta_1 = 0$ with respect to v_2, that is

$$\underline{y}\left(x\right) = \frac{3}{2} \frac{\left(6 - 13x\right)\left(2 - 3x\right)\left(2 + 5x\right)}{8 - 154x + 117x^2} \frac{\nu^0}{q^0} q_2 < 0 \; \forall \; x \in]\frac{6}{13}, \frac{2}{3}].$$

Moreover studying $\underline{y}\left(x\right)$ shows that in the interval $]\frac{6}{13}, \frac{2}{3}]$, it reaches its minimum for $x = \underline{x}$ where $\underline{y}'\left(\underline{x}\right) = 0$ for

$$\underline{x} \in \arg\left\{ x \in]\frac{6}{13}, \frac{2}{3}] \middle| 3472 - 7888x + 29824x^2 - 60060x^3 + 22\,815x^4 = 0 \right\}$$

so that $\underline{x} \simeq 0.5568$ and $\underline{y}\left(\underline{x}\right) \simeq 0.107 \frac{\nu^0}{q^0} q_2 < 0$. Therefore we can conclude that if $v_2 < \underline{y}\left(\underline{x}\right)$ we have $\partial^2 \delta_1^* / \partial a \partial \eta_1 < 0$ for all admissible x. But if $\underline{y}\left(\underline{x}\right) < v_2 < 0$, as $\partial^2 \delta_1^* / \partial a \partial \eta_1$ increases with v_2, we have $\partial^2 \delta_1^* / \partial a \partial \eta_1 > 0$ at $x = \underline{x}$. As $\underline{y}\left(x\right)$ is strictly convex in x, for each v_2 there exist two values of $\underline{x}_1$ and $\underline{x}_2$ such that $\underline{x}_2 < \underline{x} < \underline{x}_1 < \frac{6}{13}$, defined by $\underline{y}\left(\underline{x}_1\right) = \underline{y}\left(\underline{x}_2\right) = v_2$ and for which $\partial^2 \delta_1^* / \partial a \partial \eta_1 > 0$ when $x \in [\underline{x}_1, \underline{x}_2]$.

Second, assume that *A.2* holds, i.e. $v_2(p,\eta) \geq 0$ and $q_2(p,\eta) \geq 0$. In the same way as above, one finds that

$$\frac{\partial^2 \delta_2^* \left(c_0, c_0, \eta_0, \eta_0\right)}{\partial a \partial \eta_1} = -\frac{4}{3} \frac{\left(6\nu^0 - 7\theta\right)}{\left(2\nu^0 + 5\theta\right)^2} q_2 + \frac{8}{9} \frac{\left(8(\nu^0)^2 - 82\theta\nu^0 + 9\theta^2\right) q^0}{\left(2\nu^0 - 3\theta\right)\left(2\nu^0 + 5\theta\right)^3} v_2$$

Letting $\theta = xv^0$ for $x \leq \frac{2}{3}$, one can easily see that $\left(8x^2 - 82x + 9\right)\theta^2 < 0$ and $\left(6x - 7\right)\theta > 0$ if (2) holds. As a result $\partial^2 \delta_2^* / \partial a \partial \eta_1 < 0 \; \forall \; x \leq \frac{2}{3}$. ■

• **Proof of Proposition 4.** From Lemma 4 we know that around cost-based access pricing for a slight asymmetry of network 1 such that $\eta_1 > \eta_2 = \eta_0$, the critical factor is δ_1^* if $v_2(p,\eta) < 0$ and δ_2^* if $v_2(p,\eta) > 0$. If *A.1* holds, i.e. $v_2(p,\eta) < 0$, the following second order cross-partial derivative tells us how this access pricing effect is modified by an increasing network asymmetry, that is

$$\frac{\partial^2 \delta_1^* \left(c_0, c_0, \eta_0, \eta_0\right)}{\partial \eta_1 \partial a_1} = -\frac{4}{9} \frac{\left(36\left(v^0\right)^3 - 188\theta\left(v^0\right)^2 + 637\theta^2\nu^0 - 718\,\theta^3\right) q^0}{\theta\left(2\nu^0 - 3\theta\right)\left(2\nu^0 + 5\theta\right)^3} v_2$$

where $\nu^0 = v(2c_0, \eta_0)$, $q^0 = q(2c_0, \eta_0)$, $v_2 = v_2(2c_0, \eta_0)$. Let $\theta = xv^0$ for $x \leq \frac{2}{3}$. We have $(36 - 188x + 637x^2 - 718x^3) (v^0)^3 \geq 0$ and so $\frac{\partial^2 \delta_1^*(c_0, c_0, \eta_0, \eta_0)}{\partial \eta_1 \partial a_1} > 0$ if $x \geq 0.586$, and is negative otherwise. Hence in the first case of Proposition 2, we have shown that around the point of network symmetry, if $\theta > \frac{3}{5}v^0 > 0.586v^0$, then $\frac{\partial^2 \delta_1^*(c_0, c_0, \eta_0, \eta_0)}{\partial \eta_1 \partial a_1} < 0$ by $\frac{\partial \delta_1^*(c_0, c_0, \eta_0, \eta_0)}{\partial a_1} < 0$. If *A.2* holds i.e. $v_2(p, \eta) \geq 0$, the corresponding derivative of the critical discount factor δ_2^* is

$$\frac{\partial^2 \delta_2^* (c_0, c_0, \eta_0, \eta_0)}{\partial \eta_1 \partial a_1} = -\frac{4}{9} \frac{\left(36 (v^0)^3 - 132\theta (v^0)^2 + 345\theta^2 \nu^0 - 214\theta^3\right) q^0}{\theta (2\nu^0 - 3\theta) (2\nu^0 + 5\theta)^3} v_2$$

Using again the change of variables $\theta = xv^0$ for $x \leq \frac{2}{3}$ we have

$$\left(36 - 132x + 345x^2 - 214^3\right) \left(v^0\right)^3 \geq 0, \forall x,$$

and so $\frac{\partial^2 \delta_2^*(c_0, c_0, \eta_0, \eta_0)}{\partial \eta_1 \partial a_1} < 0$. From Proposition 2, we know that $\frac{\partial \delta_2^*(c_0, c_0, \eta_0, \eta_0)}{\partial a_1} \lesseqgtr 0$ as $\theta \gtreqless \frac{6}{13}\nu^0$. Then for θ 'large' we have $\frac{\partial \delta_2^*(c_0, c_0, \eta_0, \eta_0)}{\partial a_1} < 0$ and the critical discount factor is equally reduced further by (slight) network asymmetries. ■