Hotelling Competition and Political Differentiation with more than two Newspapers

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Abstract

We analyse a newspaper market where media firms compete for advertising as well as for readership. Firms first choose the political position of their newspaper, then set cover prices and advertising tariffs. We build on the duopoly work in two-sided markets of Gabszewicz, Laussel, and Sonnac (2001, 2002) who show that advertising financing can lead to minimum political differentiation. We extend their model to more than two firms and show that concerns for the emergence of a Pensée Unique as a result of advertising financing increase as the number of firms increases. In a simulation exercise we derive equilibrium locations and the welfare implications of an asymmetric shock as motivated by the empirical findings in Behringer and Filistrucchi (2010b).

1 Introduction

While the issue of endogenous product positioning in two-sided markets is interesting per se, concerns about *political pluralism* imply that product differentiation (e.g. the choice of political content covered) plays an even more crucial role in media markets than in standard markets. This is the case because of the positive externality pluralism has in the political process. As a result public concern about the role and effect of advertising on content in media markets

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is much more pronounced and the two-sided market structure of this industry deserves special attention.

Previous theoretical work has modelled spacial competition for newspapers as taking place on the political line. The work on duopoly of Gabszewicz, Laussel, and Sonnac (2001, 2002) develops a model of oligopolistic competition among media firms who choose first political position, then cover prices and advertising tariffs. In most models the choice of content is assumed to be exogenous. They show that advertising financing can lead to minimum product differentiation (Pensée Unique).

Our model features more than two oligopolistic competitors. Choosing two firms has substantial analytical advantages due to the implied complete symmetry of the firms. Having more than two firms on the political line implies that some firms may be close to the center of the political spectrum with two immediate competitors but two firms are the boundary only facing one. Hence with more than two firms individual firms are asymmetric in their competitive position preventing most analytical shortcuts that could otherwise be applied.

We find that despite this competitive asymmetry critical results of Gabszewicz, Laussel, and Sonnac and hence its policy implications can be generalized. We can conclude that concerns for minimum product differentiation as a result of advertising financing increase as the number of firms increases but that an alternative form of condensed content may appear in the four firm case where pairs of firms offer identical content. We calculate equilibrium outcomes for an asymmetric advertising shock and discuss the implied welfare consequences as motivated by our work on the UK newspaper industry in Behringer & Filistrucchi (2009b).

2 The Model

In our model product differentiation is assumed to be one dimensional and firms can charge different prices influencing the position of marginal consumers drawn from some distribution function over the characteristic space which we simplify to be the real line. The standard 'transport cost' parameterized by t is thus a shared disutility that occurs if a reader does not consume the newspaper that exactly corresponds to her most preferred variety.

Mathematically the model can become very complex once locations of the firms on the characteristic space are no longer fixed (e.g. see Anderson, (1992), p.284). Also almost all theoretical models assume that there are only *two firms* and if an n>2 firm case is analyzed symmetry assumptions about substitution patterns (e.g. the logit model) or the use of a circular characteristic space (e.g.

the Salop model) erase many of the realistic properties of the original model for the newspaper industry. An exception to this tendency is recent work by Brenner (2005) who also relaxes the duopoly restriction. Following the possible non-existence of Pure Strategy Nash equilibria (PSNE) in price for given and close locations noted by d'Aspremont, Gabszewicz, & Thisse (1979) we assume that 'transport costs' are quadratic.

A full characterization of the theoretical setup with more than two firms, variable location, variable price, and endpoints in a two-sided market implies a non-trivial analytical challenge. In order to meet this challenge we have to put some structure on the order of the player's moves, on the shape of the demand function, and on the demand for advertising.

We assume that firms play a non-cooperative *two-stage game* in which in the first stage they simultaneously chose their optimal locations on the political line and in the second stage they simultaneously chose both prices and advertising rates. The solution concept is pure strategy subgame-perfect Nash Equilibrium (SPNE) in pure strategies.

To better motivate the analysis assume that the four firms are newspapers: the Guardian (G), the Independent (I), The Times (T), and the Daily Telegraph (DT) and are differentiated on the political unit-line from Left to Right according to common consensus.

As total demand for newspapers changes over the period under investigation we generalize the standard Hotelling model allowing for *elastic demand*. In this we extend work on duopolistic settings by Böckem (1994) who allows for individually distributed reservation prices (independent of the distribution of consumers) and Hinloopen and van Marrewijk (1999) who look at common reservation prices within the linear transport cost analysis of Economides (1986).

2.1 The readers side

Consumers have utility $R - \alpha.p_{.}^{N}$ from consuming the good where $R_{.} \geq 0$ is a reservation price. They also face a quadratic transport cost that is proportional to the distance between their (political) location and that of the firm $l_{.}$ so that utilities functions of a *consumer* j who is located at x and may purchase a newspaper with quantity $q_{j,i} \in \{0, 1\}$ from *firm* i with location l_i are given by

$$u_{ji} = \left\{ \begin{array}{c} 0 \text{ if } q_{j,i} = 0 \\ R_{j,i} - \alpha_{j,i} p_i^N - t(x - l_i)^2 \text{ if } q_{j,i} = 1 \end{array} \right. \tag{1}$$

where $\alpha_{j,i}$ ($\equiv \alpha_i$) gives a marginal disutility of price for consumer j.

The position of a marginal consumer ($x_{..}$) between two firms i and $i+1$ can be determined by the indifference condition

$$R_{j,i} - \alpha_i p_i^N - t(x_{i,i+1} - l_i)^2 = R_{j,i+1} - \alpha_{i+1} p_{i+1}^N - t(x_{i,i+1} - l_{i+1})^2 \tag{2}$$

of the consumer j located at $x_{i,i+1}$. As the reservation price will not be specific to consumption of a particular good we assume that $R_{j,i} = R_{j,i+1} \forall j, i$ but the reservation price will determine whether a consumer purchases a newspaper at all. Hence a consumer located at $x \in [l_i, l_{i+1}]$ solves

$$\max \left\{ R_{j,i} - \alpha_i p_i^N - t(x - l_i)^2, R_{j,i+1} - \alpha_{i+1} p_{i+1}^N - t(x - l_{i+1})^2, 0 \right\} \tag{3}$$

The marginal consumer, denoted $x_{i,i+1}$ who is just indifferent of buying good i and $i+1$ is located at

$$x_{i,i+1} = \frac{1}{2} \frac{\alpha_{i+1} p_{i+1}^N + t l_{i+1}^2 - \alpha_i p_i^N - t l_i^2}{t\left(l_{i+1} - l_i\right)} \tag{4}$$

Clearly a positive purchase decision from (3) of the marginal consumer $x = x_{i,i+1}$ implies a positive purchase from any consumers in its neighbourhood. Symmetrically the marginal consumer on his left is located at

$$x_{i-1,i} = \frac{1}{2} \frac{\alpha_i p_i^N + t l_i^2 - p_{i-1}^N \alpha_{i-1} - t l_{i-1}^2}{t\left(l_i - l_{i-1}\right)} \tag{5}$$

Clearly $R_{j,i} = R_{j,i+1} \forall j, i \Rightarrow R_{j,i-1} = R_{j,i}$ and thus both indifference locations are independent of reservation prices.

As no consumers can be forced to purchase, *individual rationality* of the marginal consumer ($u_{ji} \geq 0 \forall j, i$) implies that reservation prices for positive purchases have to satisfy

$$R_{j,i} = R_{j,i+1} \geq \min\left\{ \alpha_i p_i^N + t(x_{i,i+1} - l_i)^2, \alpha_{i+1} p_{i+1}^N + t(x_{i,i+1} - l_{i+1})^2 \right\} \forall j, i \tag{6}$$

from which it follows that all locations for a purchase from i need to satisfy

$$x \leq l_i + \left(\frac{R_{j,i} - \alpha_i p_i^N}{t} \right)^{\frac{1}{2}} \tag{7}$$

and

$$x \geq l_i - \left(\frac{R_{j,i} - \alpha_{i1} p_i^N}{t} \right)^{\frac{1}{2}} \tag{8}$$

Similarly a purchase from $i+1$ needs consumer locations to satisfy

$$l_{i+1} - \left(\frac{R_{j,i+1} - \alpha_{i+1} p_{i+1}^N}{t} \right)^{\frac{1}{2}} \leq x \leq l_{i+1} + \left(\frac{R_{j,i+1} - \alpha_{i+1} p_{i+1}^N}{t} \right)^{\frac{1}{2}} \tag{9}$$

Given $R_{j,i}(\equiv R)$, i.e. a common reservation price the market share of some *interior* newspaper $i = I, T$ with *uniformly* distributed consumers is

$$ms_i = z_{i,i+1} - z_{i-1,i} = \min\left\{z^{NB}_{i,i+1}, z^{B}_{i,i+1}\right\} - \max\left\{z^{NB}_{i-1,i}, z^{B}_{i-1,i}\right\} \equiv \quad (10)$$
$$\min\left\{x_{i,i+1}, l_i + \left(\frac{R-\alpha_i p_i^N}{t}\right)^{\frac{1}{2}}\right\} - \max\left\{x_{i-1,i}, l_i - \left(\frac{R-\alpha_i p_i^N}{t}\right)^{\frac{1}{2}}\right\}$$

Given that reservation prices $R_{.}$ are sufficiently large so that they are *not binding* we find

$$ms_i = z^{NB}_{i,i+1} - z^{NB}_{i-1,i} = x_{i,i+1} - x_{i-1,i} = \quad (11)$$
$$\frac{1}{2t}(\alpha_{i+1}\frac{p^N_{i+1}}{l_{i+1}-l_i} - \alpha_i p_i^N \frac{l_{i+1}-l_{i-1}}{(l_{i+1}-l_i)(l_i-l_{i-1})} + \alpha_{i-1}\frac{p^N_{i-1}}{l_i-l_{i-1}} + t\,(l_{i+1}-l_{i-1}))$$

Otherwise, if *both* reservation prices bind we have

$$ms_i = z^B_{i,i+1} - z^B_{i-1,i} = l_i + \left(\frac{R-\alpha_i p_i^N}{t}\right)^{\frac{1}{2}} - \left(l_i - \left(\frac{R-\alpha_i p_i^N}{t}\right)^{\frac{1}{2}}\right) = 2\left(\frac{R-\alpha_i p_i^N}{t}\right)^{\frac{1}{2}} \quad (12)$$

which is independent of the other firms' behaviour.

If only *one* reservation price binds we either have

$$ms_i = z^B_{i,i+1} - z^{NB}_{i-1,i} = l_i + \left(\frac{R-\alpha_i p_i^N}{t}\right)^{\frac{1}{2}} - \frac{1}{2}\frac{\alpha_i p_i^N + t l_i^2 - p^N_{i-1}\alpha_{i-1} - t l^2_{i-1}}{t\,(l_i - l_{i-1})} \quad (13)$$

or

$$ms_i = z^{NB}_{i,i+1} - z^B_{i-1,i} = \frac{1}{2}\frac{\alpha_{i+1} p^N_{i+1} + t l^2_{i+1} - \alpha_i p_i^N - t l_i^2}{t\,(l_{i+1}-l_i)} - \left(l_i - \left(\frac{R-\alpha_i p_i^N}{t}\right)^{\frac{1}{2}}\right) \quad (14)$$

Note that, ceteris paribus, the property that own-price increases will decrease and other-price increases will increase own market shares will not depend on whether reservation prices are binding or not. Hence the quasiconcavity of the profit function will be preserved.

If we look at *non-interior firms,* for the LHS firm G, setting $z_{i-1,i} = \max\left\{0, l_i - \left(\frac{R-\alpha_i p_i^N}{t}\right)^{\frac{1}{2}}\right\}$ we have for sufficiently large R that

$$ms_{G=i} = z^{NB}_{i,i+1} - z^{NB}_{i-1,i} = x_{i,i+1} =$$
$$\frac{1}{2}\frac{\alpha_{i+1} p^N_{i+1} + t l^2_{i+1} - \alpha_i p_i^N - t l_i^2}{t\,(l_{i+1}-l_i)} = \frac{l_G + l_I}{2} + \frac{\alpha_I p_I^N - \alpha_G p_G^N}{2t\,(l_I - l_G)} \quad (15)$$

If reservation prices are binding for the non-interior firms they may also bind on either side (see Böckem (1994)). If only *one* reservation price binds we may thus either have

$$ms_{G=i} = z^{B}_{i,i+1} - z^{NB}_{i-1,i} = l_G + \left(\frac{R - \alpha_G p^N_G}{t}\right)^{\frac{1}{2}} - 0 \tag{16}$$

or

$$ms_{G=i} = z^{NB}_{i,i+1} - z^{B}_{i-1,i} = \frac{1}{2}\frac{\alpha_I p^N_I + tl^2_I - \alpha_G p^N_G - tl^2_G}{t\left(l_I - l_G\right)} - \left(l_G - \left(\frac{R - \alpha_G p^N_G}{t}\right)^{\frac{1}{2}}\right) \tag{17}$$

If both are binding we again have the same result (12) as with interior firms.

For the RHS firm DT, setting $z_{i,i+1} = \min\left\{1, l_i + \left(\frac{R-\alpha_i p^N_i}{t}\right)^{\frac{1}{2}}\right\}$ we have for sufficiently large R. that

$$\begin{aligned} &ms_{DT=i} = z_{i,i+1} - z_{i-1,i} = 1 - x_{i-1,i} = \\ &1 - \frac{1}{2}\frac{\alpha_i p^N_i + tl^2_i - p^N_{i-1}\alpha_{i-1} - tl^2_{i-1}}{t\left(l_i - l_{i-1}\right)} = 1 - \left(\frac{l_T + l_{DT}}{2} + \frac{\alpha_{DT} p^N_{DT} - \alpha_T p^N_T}{2t\left(l_{DT} - l_T\right)}\right) \end{aligned} \tag{18}$$

again if *one* reservation price binds we either have

$$ms_{DT=i} = z^{NB}_{i,i+1} - z^{B}_{i-1,i} = 1 - x_{i-1,i} = 1 - \left(l_{DT} - \left(\frac{R - \alpha_{DT} p^N_{DT}}{t}\right)^{\frac{1}{2}}\right) \tag{19}$$

or

$$ms_{DT=i} = z^{B}_{i,i+1} - z^{NB}_{i-1,i} = l_{DT} + \left(\frac{R - \alpha_{DT} p^N_{DT}}{t}\right)^{\frac{1}{2}} - \frac{1}{2}\frac{\alpha_{DT} p^N_{DT} + tl^2_{DT} - p^N_T \alpha_T - tl^2_T}{t\left(l_{DT} - l_T\right)} \tag{20}$$

and if are binding we again have the same result (12) as with interior firms.

If we further assume $\alpha_i = \alpha\ \forall i$ the non-binding system reduces to a demand for an interior firm $i = I, T$ of

$$ms_i = \frac{l_{i+1} - l_{i-1}}{2} + \frac{\alpha}{t}\frac{p^N_{i+1} - p^N_i}{2\left(l_{i+1} - l_i\right)} - \frac{\alpha}{t}\frac{p^N_i - p^N_{i-1}}{2\left(l_i - l_{i-1}\right)} \tag{21}$$

and that of the firm G on the LHS as

$$ms_G = \frac{l_G + l_I}{2} + \frac{\alpha}{t}\frac{p^N_I - p^N_G}{2\left(l_T - l_G\right)} \tag{22}$$

and that for DT on the RHS as

$$ms_{DT} = 1 - \left(\frac{l_T + l_{DT}}{2} + \frac{\alpha}{t}\frac{p^N_{DT} - p^N_T}{2\left(l_{DT} - l_T\right)}\right) \tag{23}$$

The resulting elasticity matrix given the 4 newspapers is

$$\begin{vmatrix} \varepsilon_{G,G} & \varepsilon_{G,I} & \varepsilon_{G,T} & \varepsilon_{G,DT} \\ \varepsilon_{I,G} & \varepsilon_{I,I} & \varepsilon_{I,T} & \varepsilon_{I,DT} \\ \varepsilon_{T,G} & \varepsilon_{T,I} & \varepsilon_{T,T} & \varepsilon_{T,DT} \\ \varepsilon_{DT,G} & \varepsilon_{DT,I} & \varepsilon_{DT,T} & \varepsilon_{DT,DT} \end{vmatrix} = \tag{24}$$

$$\frac{\alpha}{2t} \begin{vmatrix} -\frac{1}{(l_I - l_G)} \frac{p_G^N}{ms_G} & \frac{1}{(l_I - l_G)} \frac{p_I^N}{ms_G} & 0 & 0 \\ \frac{1}{(l_I - l_G)} \frac{p_G^N}{ms_I} & -\frac{l_T - l_G}{(l_T - l_I)(l_I - l_G)} \frac{p_I^N}{ms_I} & \frac{1}{(l_T - l_I)} \frac{p_T^N}{ms_I} & 0 \\ 0 & \frac{1}{(l_T - l_I)} \frac{p_I^N}{ms_T} & -\frac{l_{DT} - l_I}{(l_{DT} - l_T)(l_T - l_I)} \frac{p_T^N}{ms_T} & \frac{1}{(l_{DT} - l_T)} \frac{p_{DT}^N}{ms_T} \\ 0 & 0 & \frac{1}{(l_{DT} - l_T)} \frac{p_T^N}{ms_{DT}} & -\frac{1}{(l_{DT} - l_T)} \frac{p_{DT}^N}{ms_{DT}} \end{vmatrix}$$

Note that these elasticities depend on locations, i.e. the newspaper's optimal choices in the first stage of the game.

Locations l, newspaper prices p^N, and advertising rates p^A are determined in a non-cooperative supply side game. Thus we determine equilibrium prices $p^*(\mathbf{l})$ and r^* given the location vector in stage II and then the SPNE location vector $\mathbf{l}^* = (l_G^*, l_I^*, l_T^*, l_{DT}^*)'$ in stage I.

2.2 The advertising side

Profit also depend on the other side of the market, i.e. on advertising revenue. Hence firms also simultaneously chose optimal advertising rates p_i^A.

As in the model of Rysman (2004), advertising utility will depend positively on newspapers share of the readers market. Gabszewicz, Laussel, and Sonnac (2002) look at the political differentiation of two newspapers in a stage game model and make an extension to the classical model of *vertical product differentiation* (as in Gabszewicz & Thisse (1979) or Shaked & Sutton (1983)) by allowing for *multiple purchases* of advertisers. We extend their analysis of newspaper advertising to $I > 2$ two firms.

An advertiser k has a benefit from advertising in newspaper i given by

$$u_k(i) = ms_i^N \theta - p_i^A \tag{25}$$

where $\theta \in [0, 1]$ gives the advertiser's intensity (or ability) of preference of buying the advert. Assume that the θ are also distributed uniformly on the unit line with density $4\phi > 0$. If the advertiser purchases from any $m > 1$ newspapers then his utility is simply $\sum^m u_k(m)$.

The advertiser will thus buy an advert from newspaper i rather than none (participation condition) if

$$u_k(i) \geq 0 \Leftrightarrow \theta \geq \frac{p_i^A}{ms_i^N} \tag{26}$$

so that only the per-reader advertising rate matters. Without loss of generality order all newspapers by their reader market shares as

$$ms_1^N > ... > ms_{i-1}^N > ms_i^N > ms_{i+1}^N > ... > ms_I^N \tag{27}$$

Now k will prefer advertising with $h \in \{1, ..., i-1, i, i+1, ...I-1\}$ to any other $h' \in \{j+1, ...I\}$ if and only if his type satisfies

$$u_k(h) \geq u_k(h') \Leftrightarrow \theta \geq \frac{p_h^A - p_{h'}^A}{ms_h^N - ms_{h'}^N} \tag{28}$$

Note that by the ordering denominators are strictly positive so that $p_{h'}^A > p_h^A$ implies that this condition holds for all $\theta \in [0, 1]$ and numerator and denominators cannot be negative at the same time. If (28) is strictly binding this is the indifference condition of the classical model.

An important property that follows from the additive utility structure of multiple purchases for any $m > 1$ newspapers is that

$$\sum^{m} u_k(m) + u_k(i) \geq \sum^{m} u_k(m) \Leftrightarrow u_k(i) \geq 0 \tag{29}$$

so that the additional purchase is beneficial if and only if the participation condition (a degenerate indifference condition) for the additional advertising holds.

Whence one has to decide whether a satisfaction of this condition on the unit line implies that only this advertising is purchased or whether it is purchased with other advertising in a multiple purchase. The possibility of multiple purchases implies that the indifference condition of the classical model has no influence on market shares in this model that will be determined by the participation constraints only but it will influence the *ordering* of these participation conditions. We can show the following:

Lemma 1 *The vertical differentiation model with multiple purchases of Gabszewicz, Laussel, and Sonnac (2002) extends to $I > 2$ firms and leads to advertising demand as*

$$ms_i^A = ms_i^A(\mathbf{p}^A, \mathbf{ms}^N(\mathbf{p}^N(\mathbf{l}))) = 4\phi\left(1 - \frac{p_i^A}{ms_i^N(\mathbf{p}^N(\mathbf{l}))}\right) \tag{30}$$

with the Nash equilibrium in advertising rates necessarily satisfying

$$p_i^{A*} = \frac{ms_i^N(\mathbf{p}^N(\mathbf{l}))}{2} \;\forall\; i \in \{1, ..., I\} \tag{31}$$

Proof:

As emphasized by Gabszewicz, Laussel, and Sonnac (2002) with multiple purchases the indifference condition (given by the critical type for which (28) is strictly binding) is important to determine *which* advertising is purchased as

$$D_{h,h'} \equiv \frac{p_h^A - p_{h'}^A}{ms_h^N - ms_{h'}^N} \leq \frac{p_i^A}{ms_i^N} \text{ for any } i = h, h' \Leftrightarrow \frac{p_h^A}{ms_h^N} \leq \frac{p_{h'}^A}{ms_{h'}^N} \tag{32}$$

and conversely

$$D_{h,h'} \equiv \frac{p_h^A - p_{h'}^A}{ms_h^N - ms_{h'}^N} \geq \frac{p_i^A}{ms_i^N} \text{ for any } i = h, h' \Leftrightarrow \frac{p_h^A}{ms_h^N} \geq \frac{p_{h'}^A}{ms_{h'}^N} \tag{33}$$

These conditions imply that an indifference condition for any two advertisements cannot be in between the two respective participation conditions.

In our setting with I firms and multiple purchases there are $(I^2 - I)/2$ indifference conditions that, for any given prices $p_i^A \in \mathbb{R}^+$, may fall anywhere on the θ-unit line thus determining the relative position of the two participation conditions concerned. A unique final ordering of participation conditions does not require knowledge of all indifference conditions however.

For example if $I = 4$ then

$$u_k(1) > u_k(2) \Leftrightarrow \theta > D_{1,2} \equiv \frac{p_1^A - p_2^A}{ms_1^N - ms_2^N} \tag{34}$$

To transform the ordering of market shares to that of the participation conditions on the θ-unit line we need to have

$$D_{1,2} < \frac{p_1^A}{ms_1^N} < \frac{p_2^A}{ms_2^N} \wedge D_{1,3} < \frac{p_1^A}{ms_1^N} < \frac{p_3^A}{ms_3^N} \wedge D_{1,4} < \frac{p_1^A}{ms_1^N} < \frac{p_4^A}{ms_4^N} \tag{35}$$

so that we need

$$D_{1,.} < \frac{p_1^A}{ms_1^N} < Min(\frac{p_{h=2,3,4}^A}{ms_{h=2,3,4}^N}) \tag{36}$$

By the same reasoning in order to order the participation conditions for firm 2 we need that

$$D_{2,3} < \frac{p_2^A}{ms_2^N} < \frac{p_3^A}{ms_3^N} \wedge D_{2,4} < \frac{p_2^A}{ms_2^N} < \frac{p_4^A}{ms_4^N} \tag{37}$$

or

$$D_{2,.} < \frac{p_2^A}{ms_2^N} < Min(\frac{p_{h=3,4}^A}{ms_{h=3,4}^N}) \tag{38}$$

Eventually for firm 3 we need

$$D_{3,4} < \frac{p_3^A}{ms_3^N} < \frac{p_4^A}{ms_4^N} \tag{39}$$

Note that these conditions unravel backwards so that we only need *three* conditions, $D_{3,4}, D_{2,3}$, and $D_{1,2}$ to determine a full ordering of advertiser types.

If we also have that $D_{1,2} < D_{2,3} < D_{3,4}$ (and the other $D_{.,.}$ located such that transitivity is satisfied for any $\theta \in [0, 1]$) this implies that advertisers will purchase from $\{\oslash; (1); (1, 2); (1, 2, 3); (1, 2, 3, 4)\}$ whenever the respective participation condition holds. In the classical model with two firms and a single purchase, $D_{1,2} < \frac{p_1^A}{ms_1^N} < \frac{p_2^A}{ms_2^N}$ implies that this set is $\{\oslash; (1)\}$ so that firm 2 has no sales. If $D_{1,2} > \frac{p_1^A}{ms_1^N} > \frac{p_2^A}{ms_2^N}$ this set is $\{\oslash; (1); (2)\}$ and market shares are determined by participation and the indifference condition. Hence there is no general demand form.

With multiple purchases the final locations of $D_{.,.}$ will *not* affect market shares and hence demand for advertising can always be written as

$$ms_i^A = ms_i^A(\mathbf{p}^A, \mathbf{ms}^N(\mathbf{p}^N(\mathbf{l}))) = 4\phi \left(1 - \frac{p_i^A}{ms_i^N(\mathbf{p}^N(\mathbf{l}))}\right) \tag{40}$$

The decision to advertise with one firm will be independent of rates and circulations of the other newspapers so that profit maximization of the advertising profit yields

$$p_i^{A*} = \frac{ms_i^N(\mathbf{p}^N(\mathbf{l}))}{2} \forall\ i \in \{1, ..., I\} \tag{41}$$

.■

2.3 Solving stage II

Profits with differentiated products in *stage II* are

$$\pi_i^{II}(p_i^A, p_i^N) = ms_i^N(\mathbf{p}^N(\mathbf{l}))(p_i^N(\mathbf{l}) - c_i^N) + ms_i^A(\mathbf{p}^A, \mathbf{ms}^N(\mathbf{p}^N(\mathbf{l})))p_i^A \tag{42}$$

As the newspaper industry is a prima facie case of a two-sided market there is a second side to the firm's overall profit that results from sales of advertising slots to advertisers. Advertising demand for a particular newspaper will increase in the newspaper's reach and hence its reader market share.

Lemma 2 *The Nash equilibrium in prices necessarily satisfies*

$$\frac{p_i^N(\mathbf{l}) - c_i^N}{p_i^N(\mathbf{l})} = -\frac{1}{\varepsilon_{ii}} \left(1 + \frac{p_i^A}{p_i^N(\mathbf{l})} \left(\frac{\partial ms_i^A}{\partial ms_i^N}\varepsilon_{ii} + \frac{1}{ms_i^N} \sum_{h \neq i} \frac{\partial ms_i^A}{\partial ms_h^N}\varepsilon_{hi} ms_h^N\right)\right) \tag{43}$$

where $\varepsilon_{ii} = \frac{\partial ms_i^N}{\partial p_i^N(\mathbf{l})} \frac{p_i^N(\mathbf{l})}{ms_i^N(\mathbf{p}^N(\mathbf{l}))}$ *and* $\varepsilon_{hi} = \frac{\partial ms_h^N}{\partial p_i^N(\mathbf{l})} \frac{p_i^N(\mathbf{l})}{ms_h^N(\mathbf{p}^N(\mathbf{l}))}$ *are respectively own- and cross-price elasticities.*

Proof: Take derivative.■

Given the Hotelling demand and elasticity structure the markup

$$\frac{p_i^N(\mathbf{l}) - c_i^N}{p_i^N(\mathbf{l})} = -\frac{1}{\varepsilon_{ii}} \left(1 + \frac{p_i^A}{p_i^N(\mathbf{l})} \left(\frac{\partial ms_i^A}{\partial ms_i^N}\varepsilon_{ii} + \frac{1}{ms_i^N} \sum_{h \neq i} \frac{\partial ms_i^A}{\partial ms_h^N}\varepsilon_{hi} ms_h^N\right)\right) \tag{44}$$

for example for the Independent, using ε_{II}; ε_{GI}, and ε_{TI} simplifies to

$$p_I^N(\mathbf{1}) - c_I^N = \frac{2ms_I^N\,(l_T - l_I)\,(l_I - l_G)\,t}{(l_T - l_G)\alpha} \times$$
$$\left(1 + \frac{p_I^A}{ms_I^N}\left(\begin{array}{c} \frac{\partial ms_I^A}{\partial ms_I^N}\left(-\frac{\alpha(l_T - l_G)}{t(l_T - l_I)(l_I - l_G)}\right) + \\ \frac{\partial ms_I^A}{\partial ms_G^N}\left(\frac{\alpha}{2t(l_I - l_G)}\right) + \frac{\partial ms_I^A}{\partial ms_T^N}\left(\frac{\alpha}{2t(l_T - l_I)}\right)\end{array}\right)\right) \tag{45}$$

Solving for the equilibrium prices of the second stage closed-form solutions are possible.

2.3.1 Equilibrium prices with non-binding reservation constraints

Given the market shares from (10) and some simplifications we find that equilibrium prices for the Independent are

$$p_I^N(\mathbf{1}) = \frac{1}{2}c_I^N + \frac{(l_T - l_I)\,(l_I - l_G)\,t}{(l_T - l_G)\alpha}\left(\frac{l_T - l_G}{2} + \frac{\alpha p_T^N(\mathbf{1})}{2t\,(l_T - l_I)} + \frac{\alpha p_G^N(\mathbf{1})}{2t\,(l_I - l_G)}\right) - \frac{1}{2}A_I \tag{46}$$

where $A_I \equiv p_I^A \frac{\partial ms_I^A}{\partial ms_I^N}$. By symmetry for the Times we have

$$p_T^N(\mathbf{1}) = \frac{1}{2}c_T^N + \frac{(l_{DT} - l_T)\,(l_T - l_I)\,t}{(l_{DT} - l_I)\alpha}\left(\frac{l_{DT} - l_I}{2} + \frac{\alpha p_{DT}^N(\mathbf{1})}{2t\,(l_{DT} - l_T)} + \frac{\alpha p_I^N(\mathbf{1})}{2t\,(l_T - l_I)}\right) - \frac{1}{2}A_T \tag{47}$$

For the Guardian, using ε_{GG} (44) simplifies to

$$\begin{aligned} p_G^N(\mathbf{1}) - c_G^N &= 2\frac{t}{\alpha}\,(l_I - l_G)\,ms_G - A_i = \\ & 2\frac{t}{\alpha}\,(l_I - l_G)\left(\frac{l_G + l_I}{2} + \frac{p_I^N(\mathbf{1}) - p_G^N(\mathbf{1})}{2t\,(l_I - l_G)}\right) - A_G \end{aligned} \tag{48}$$

which we can solve for prices as

$$p_G^N(\mathbf{1}) = \frac{1}{2}c_G^N + \frac{1}{2}\,(l_I - l_G)\,(l_G + l_I)\,\frac{t}{\alpha} + \frac{1}{2}p_I^N(\mathbf{1}) - \frac{1}{2}A_G \tag{49}$$

For the Daily Telegraph, using $\varepsilon_{DT,DT}$ we have

$$\begin{aligned} p_{DT}^N(\mathbf{1}) - c_{DT}^N &= 2\frac{t}{\alpha}\,(l_{DT} - l_T)\,ms_{DT} - A_{DT} = \\ & 2\frac{t}{\alpha}\,(l_{DT} - l_T)\left(1 - \left(\frac{l_T + l_{DT}}{2} + \frac{\alpha(p_{DT}^N(\mathbf{1}) - p_T^N(\mathbf{1}))}{2t\,(l_{DT} - l_T)}\right)\right) - A_{DT} \end{aligned} \tag{50}$$

which can be solved as

$$p_{DT}^{N}(\mathbf{l}) = \frac{1}{2}c_{DT}^{N} + l_{DT}(1-\frac{1}{2}l_{DT})\frac{t}{\alpha} - l_{T}(1-\frac{1}{2}l_{T})\frac{t}{\alpha} + \frac{1}{2}p_{T}^{N}(\mathbf{l}) - \frac{1}{2}A_{DT} \quad (51)$$

We thus have as system of reaction functions of the price game consisting of 4 equations in 4 unknowns as

$$p_{G}^{N} = \max\left\{0, \frac{1}{2}c_{G}^{N} + \frac{1}{2}\frac{t}{\alpha}\left(l_{I}-l_{G}\right)\left(l_{G}+l_{I}\right) + \frac{1}{2}p_{I}^{N} - \frac{1}{2}A_{G}\right\} \quad (52)$$

$$p_{I}^{N} = \max\left\{0, \frac{1}{2}c_{I}^{N} + \frac{\left(l_{T}-l_{I}\right)\left(l_{I}-l_{G}\right)t}{(l_{T}-l_{G})\alpha}\left(\frac{l_{T}-l_{G}}{2} + \frac{\alpha p_{T}^{N}}{2t\left(l_{T}-l_{I}\right)} + \frac{\alpha p_{G}^{N}}{2t\left(l_{I}-l_{G}\right)}\right) - \frac{1}{2}A_{I}\right\} \quad (53)$$

$$p_{T}^{N} = \max\left\{0, \frac{1}{2}c_{T}^{N} + \frac{\left(l_{DT}-l_{T}\right)\left(l_{T}-l_{I}\right)t}{(l_{DT}-l_{I})\alpha}\left(\frac{l_{DT}-l_{I}}{2} + \frac{\alpha p_{DT}^{N}}{2t\left(l_{DT}-l_{T}\right)} + \frac{\alpha p_{I}^{N}}{2t\left(l_{T}-l_{I}\right)}\right) - \frac{1}{2}A_{T}\right\} \quad \text{(54a)}$$

$$p_{DT}^{N} = \max\left\{0, \frac{1}{2}c_{DT}^{N} + l_{DT}(1-\frac{1}{2}l_{DT})\frac{t}{\alpha} - l_{T}(1-\frac{1}{2}l_{T})\frac{t}{\alpha} + \frac{1}{2}p_{T}^{N} - \frac{1}{2}A_{DT}\right\} \quad (55)$$

For all prices to be 0 simultaneously we need to be in a region where (by adding up the 4 inequality constraints that result if the RHS of the brackets above are less than zero for zero prices):

$$\begin{array}{c} A_{DT} - c_{DT}^{N} + A_{T} - c_{T}^{N} + A_{I} - c_{I}^{N} + A_{G} - c_{G}^{N} + \\ \frac{t}{\alpha}\left(\left(l_{DT}-l_{T}+l_{I}-2\right)l_{DT} + l_{G}^{2} + \left(l_{T}-l_{I}\right)l_{G} + 2l_{T}\left(1-l_{I}\right)\right) \end{array} > 0 \quad (56)$$

i.e. if advertising is very important for profits.

Note that in equilibrium

$$A_{i}^{*} = p_{i}^{A*}\frac{\partial ms_{i}^{A*}}{\partial ms_{i}^{N}} = \frac{ms_{i}^{N}(\mathbf{p}^{N}(\mathbf{l}))}{2}4\frac{\phi}{(ms_{i}^{N}(\mathbf{p}^{N}(\mathbf{l}))^{2}}p_{i}^{A*} = \phi \quad (57)$$

In matrix form define this system of equations as

$$\frac{1}{2}\mathbf{A} = \mathbf{\Omega}\mathbf{p}^{N*}(\mathbf{l}) \quad (58)$$

where we write equations (46), (47), (49), and (51) as:

$$\frac{1}{2}\left|\begin{array}{c} -c_G^N - \frac{t}{\alpha}\left(l_I - l_G\right)\left(l_G + l_I\right) + A_G \\ -c_I^N - \frac{t}{\alpha}\left(l_T - l_I\right)\left(l_I - l_G\right) + A_I \\ -c_T^N - \frac{t}{\alpha}\left(l_{DT} - l_T\right)\left(l_T - l_I\right) + A_T \\ -c_{DT}^N - 2l_{DT}(1 - \frac{1}{2}l_{DT})\frac{t}{\alpha} + 2l_T(1 - \frac{1}{2}l_T)\frac{t}{\alpha} + A_{DT} \end{array}\right| =$$

$$\left|\begin{array}{cccc} -1 & \frac{1}{2} & 0 & 0 \\ \frac{l_T - l_I}{2(l_T - l_G)} & -1 & \frac{l_I - l_G}{2(l_T - l_G)} & 0 \\ 0 & \frac{l_{DT} - l_T}{2(l_{DT} - l_I)} & -1 & \frac{l_T - l_I}{2(l_{DT} - l_I)} \\ 0 & 0 & \frac{1}{2} & -1 \end{array}\right| x \left|\begin{array}{c} p_G^N \\ p_I^N \\ p_T^N \\ p_{DT}^N \end{array}\right| \tag{59}$$

which can be solved analytically for the equilibrium price vector $\mathbf{p}^{N*}(\mathbf{l})' = (p_G^{N*}(\mathbf{l}), p_I^{N*}(\mathbf{l}), p_T^{N*}(\mathbf{l}), p_{DT}^{N*}(\mathbf{l}))'$ by inverting the system as.

Lemma 3 *In the case of a non-binding reservation constraint*

$$\frac{\partial p_i^{N*}(\mathbf{l})}{\partial A} = -1 \textit{ where } A_i = A \ \forall i = G, I, T, DT$$

Proof:
See Appendix.■

Hence as in Gabszewicz at. al. with two firms a "full pass-through" effect prevails and an increase in the per-reader adverting revenue decreases the equilibrium newspaper prices one-to-one.

2.3.2 Equilibrium prices with binding reservation constraints

If reservation prices bind on both sides of the firms we have that for *interior and non-interior* firms

$$ms_i^N = z_{i,i+1}^B - z_{i-1,i}^B = 2\left(\frac{R - \alpha_i p_i^N}{t}\right)^{\frac{1}{2}} \tag{60}$$

which is independent of the other firms and increasing in the reservation price.

Hence it is a *dominant strategy* for all firms to set

$$p_i^{N*} = \frac{2}{3}\left(\frac{R}{\alpha_i} + \frac{1}{2}(c_i^N - A_i)\right) \ \forall i = G, I, T, DT \tag{61}$$

$$ms_i = \frac{2\sqrt{3}}{3}\sqrt{\frac{1}{t}\left(\frac{R}{\alpha_i} - c_i^N + A_i\right)} \ \forall i = G, I, T, DT \tag{62}$$

and equilibrium profits from selling to readers given by

$$\pi_i = \frac{4\sqrt{3}}{9\alpha_i}\sqrt{\frac{1}{t\alpha_i}\left(R - c_i^N\alpha_i + A_i\alpha_i\right)}(R - c_i^N\alpha_i + A_i\alpha_i)\ \forall i = G, I, T, DT \quad (63)$$

which is increasing in the reservation price (which generates more sales and increases the equilibrium price) but decreasing in t and α,the (income-) sensitivity of a price change on utility.

Lemma 4 *In the case of a binding reservation constraint*

$$\frac{\partial p_i^N}{\partial A_i} = \frac{2}{3}\sqrt{3}\sqrt{\frac{1}{t\alpha_i}\left(R + \alpha_i(A_i - c_i^N\right)} > 0\ \forall i = G, I, T, DT$$

Proof:
Take derivative.■

Hence unlike in the case with non-binding reservation constraint there is no "full-pass through effect" and an increase in the adverting demand increases the equilibrium price.

2.4 Solving stage I

Using the results from stage II of the game we can first show the following:

Proposition 5 *Given the structure of the advertising side, the problem at the first stage can be transformed into a Hotelling problem with profits depending only on location as* $\pi'(p_i^{A*}, p_i^{N*}) = (p_i^*(\mathbf{l}) - c_i')ms_i(\mathbf{p}^*(\mathbf{l}))$.

Proof:
See Appendix.■

Total profits in stage I given the equilibrium price vector $\mathbf{p}^{N*}(\mathbf{l})'$ with generic element p_i^{N*} and p_i^{A*} from stage II can then be written as

$$\pi_i(p_i^{A*}, p_i^{N*}(\mathbf{l})) = \left((p_i^{N*}(\mathbf{l}) \underbrace{-c_i^N + A_i}_{-c_i'} \right) ms_i^N(\mathbf{p}^{N*}(\mathbf{l})) \quad (64)$$

so that stage I equilibrium profits are *linear* in the reader market shares that are given by the Hotelling specification, i.e. for interior firms see (10).

3 Guaranteeing pluralism

One major finding in the work of Gabszewicz, Laussel, & Sonnac (2002) is that in order for the standard maximum differentiation result in duopoly of d'Aspremont, Gabszewicz, and Thisse (1979) to hold advertising demand cannot be too pronounced. Inversely a situation exists where the advertising demand is so important that sharing the market for readers becomes the primal objective and minimum differentiation results. Both cases can overlap.

We assume an equal sharing rule in case of minimum differentiation. A version of their respective lemma states

Lemma 6 *There exists a Nash equilibrium with full minimum differentiation of* (l_L, l_R) *in the center in the duopoly game if*

$$A - c > t$$

Proof:
Note that their l_R is $1 - l_R$ in our notation. Best response functions in the duopoly case are

$$\begin{aligned} p_L &= \max\left\{0, \frac{1}{2}(c - A + p_R + t - 2tl_R + tl_R^2, -tl_L^2)\right\} \\ p_R &= \max\left\{0, \frac{1}{2}(c - A + p_I + t - 2tl_L + tl_L^2, -tl_R^2)\right\} \end{aligned}$$

Adding up LHS conditions at zero prices we find

$$A > c + t(1 - l_R - l_L)$$

The game is then a pure location game at zero equilibrium prices for given locations. A sufficient condition for this to hold for *any* location (and hence any possible deviation) is

$$A > c + t(1 - 0 - 0)$$

so that there cannot be any deviation that leads to positive prices and hence to any other equilibrium but the PSNE in locations $l_L = l_R = \frac{1}{2}$.■

In our 4-firm case we find:

Lemma 7 *There exists a Nash equilibrium with full minimum differentiation of (l_G, l_I, l_T, l_{DT}) in the center in the quadropoly game if*

$$A - c > \frac{1}{8}t$$

Proof:
The equivalent zero-price condition was found in (56) as

$$\begin{array}{c} A_{DT} - c^N_{DT} + A_T - c^N_T + A_I - c^N_I + A_G - c^N_G + \\ \frac{t}{\alpha}\left(\left(l_{DT} - l_T + l_I - 2\right) l_{DT} + l_G^2 + \left(l_T - l_I\right) l_G + 2l_T\left(1 - l_I\right)\right) \end{array} > 0$$

Note for a symmetric costs and ads revenues we find *symmetric* locations as $l_G = 1 - l_{DT}$ and $l_I = 1 - l_T$ the condition simplifies to

$$4(A - c) + \frac{t}{\alpha}(2\left(l_T - l_{DT} + 1\right)\left(l_T - l_{TD}\right)) > 0$$

where

$$2\left(l_T - l_{DT} + 1\right)\left(l_T - l_{DT}\right) < 0$$

To make this as small as possible we aim to

$$\max_{l_T, l_{DT}} -\left(l_T - l_{DT} + 1\right)\left(l_T - l_{DT}\right) \text{ s.t. } l_{DT} \geq l_T$$

The first order conditions yield the condition

$$l_T + \frac{1}{2} = l_{DT}$$

Using symmetry and the ordering $l_T \geq l_I$ the unique solution is

$$l_T^* = \frac{1}{2}, l_{DT}^* = 1$$

The best one can do in order to guard pluralism is then to select locations

$$l_G^* = 0, l_I^* = l_T^* = \frac{1}{2}, l_{DT}^* = 1$$

for which we find the all-zero price condition (56) (normalize $\alpha = 1$) as

$$A > c + \frac{1}{8}t$$

If this condition holds then for any other symmetric location the zero price constraint binds and thus $l_G, l_I, l_T, l_{DT} = \frac{1}{2}$ is a Nash equilibrium given the sharing rule hold.■

Note that minimum differentiation is not the only symmetric pure locations equilibrium in the Hotelling 4-firm game. The alternative pure location equilibrium candidate that does not require a sharing rule to hold is $(1/4, 1/4, 3/4, 3/4)$ for which the zero-price condition similarly reduces to $A > c$, i.e. the same condition that has to hold for the minimum differentiation equilibrium.

Hence even if the zero-price outcome is more probable in the 4-firm case this need not imply that there necessarily is the tendency for minimum differentiation of *all* firms.

Most generally we find for the n-firm case (for $n \geq 4$) that:

Proposition 8 *There exists a Nash equilibrium with full minimum differentiation of* $(l_1, l_2, ..., l_{n-1}, l_n)$ *in the center in the n-firm oligopoly game if*

$$A - c > \frac{1}{2n}t$$

Proof: See Appendix.■

Note that in order to make the occurrence of minimum differentiation least likely the locations for the firms will always be $l_1 = 0$, $l_2 = ... = l_{n-2} = l_{n-1} = \frac{1}{2}, l_n = 1$. Here the $(n-2)$ firms located in the middle have a zero price and only the two extreme firms have a strictly positive one. Thus in order to best guarantee a full minimum differentiation result one chooses a partial minimum differentiation of $(n-2)$ firms and lets the remaining firms cover the extreme positions.

For completeness we report the condition for $n = 3$ which reads $A - c > \frac{1}{4}t$. This does not fit into the above general proposition as the firm in the center retains a positive price. The is no SPNE in the 3 firm pure location game.

4 Simulation

Given the model outlined above, we can simulate changes of the exogenous variables and derive equilibrium comparative statics results.

In particular Proposition 5 implies that our two-sided market setup can be reduced to a Hotelling problem with four firms, simultaneous choices of location and prices and advertising rates. A similar problem has been analysed in Götz, (2005) who extends work on the Hotelling model by Neven (1987) and Economides (1993). We thus modify his Mathematica algorithm to solve for the equilibrium of the first stage locations explicitly. This exercise has been undertaken also in Brenner (2005) for up to nine firms.

The algorithm is based on a *Newton-Raphson approximation* of the equilibrium first stage location with starting values $\mathbf{l}'$=(0,.3, .6, 1)$'$. The algorithm proceeds by evaluating the tangent on the original function at the starting value, finds its intercept with the abscissa which is then used to find the functional value and tangent again recursively until the point of tangency and the intercept with the abscissa coincide. The algorithm converges and sufficiency of the first order necessary conditions for optimality can be checked by looking at the profit function for local deviations.

4.1 The symmetric situation

We assume that $t/\alpha = 10$ and that the extended cost is $c_i^{'} = .5$ for all firms. The NR-algorithm then yields equilibrium location on the political line in the first stage of the game as

$$\mathbf{l}^* = (l_G^*, l_I^*, l_T^*, l_{DT}^*)' = (.124, .396, .604, .876)' \tag{65}$$

Note that despite the symmetry of the situation there remains a difference between the interior and the non-interior newspapers. This implies that only the locations of interior newspapers I and T and those of the non-interior newspapers G and DT are mirror images, i.e. $l_G^* = 1 - l_{DT}^*$ and $l_I^* = 1 - l_T^*$. These equilibrium locations imply equilibrium prices in the second stage as

$$\mathbf{p}^{N*}(\mathbf{l}^*) = (p_G^*(\mathbf{l}^*), p_I^*(\mathbf{l}^*), p_T^*(\mathbf{l}^*), p_{DT}^*(\mathbf{l}^*))' = (1.566, 1.216, 1.216, 1.566)' \tag{66}$$

where again we observe the symmetry between interior and non-interior firms.

The equilibrium market share vector of the readers market is $\mathbf{ms}^N(\mathbf{p}^{N*}(\mathbf{l}^*)) =$ $(.196, .304, .304, .196)'$which corresponds to the market share vector of the advertising market by (30). Equilibrium profits are

$$\boldsymbol{\pi}^I(\mathbf{p}^{A*}, \mathbf{p}^{N*}(\mathbf{l}^*)) = (.209, .218, .218, .209)' \tag{67}$$

Note that market share are still asymmetric with regard to own location of the interior and non-interior firms as

$$\begin{aligned} ms_{\neg int}R &= ms_G - l_G = .196 - .124 = .072 \\ ms_{\neg int}L &= ms_G - ms_{\neg int}R = .196 - .072 = .124 \\ ms_{int}R &= \frac{1}{2} - l_I = \frac{1}{2} - .396 = .104 \\ ms_{int}L &= ms_I - ms_{int}R = .304 - .104 = .2 \end{aligned} \tag{68}$$

Welfare costs to consumers in this case can be calculated as

$$\begin{aligned} & WC_S = \\ & 2t\begin{pmatrix} \int_0^{ms_{\neg int}L} x^2dx + \int_0^{ms_{\neg int}R} x^2dx + \\ \int_0^{ms_{int}L} x^2dx + \int_0^{ms_{int}R} x^2dx \end{pmatrix} + 2(p_{\ulcorner int}, p_{int})\begin{pmatrix} ms_{\neg int} \\ ms_{int} \end{pmatrix} = \\ & 2t\begin{pmatrix} \int_0^{0.124} x^2dx + \int_0^{0.072} x^2dx + \\ \int_0^{0.2} x^2dx + \int_0^{0.104} x^2dx \end{pmatrix} + 2(1.566, 1.216)\begin{pmatrix} .196 \\ .304 \end{pmatrix} = \\ & 7.6032 \times 10^{-3}t + 1.3532 \end{aligned} \tag{69}$$

for $t = 10$ ($\alpha = 1$) this is $WC_S = 1.4292$.

4.2 An asymmetric advertising shock

Motivation from the UK newspaper industry: As described in detail in Behringer & Filistrucchi (2010b) as a consequence of the economies of scale and scope within media market a possible consequence of Rupert Murdoch's acquisition of the Times in 1981 is an increased asymmetry between The Times and its competitors.

To explore the effect of such asymmetric changes in the competitive environment we assume that starting from a symmetric situation the marginal cost of production of the newspaper Times c_i^N decreases and/or the per-capita advertising revenue A_i increases (reducing the extended cost $c_i^{'}$). This may be due to a substantial printing cost advantage resulting from more efficient production methods at Wapping or due to some exogenous increase of the demand for advertising for The Times only (both are consistent with the Times joining a media conglomerate such as Rupert Murdoch's).

We investigate the effect of this change on the equilibrium magnitudes of the model. Assuming that the extended cost of The Times T exogenously drops to $c_i^{'} = .2$ the equilibrium magnitudes change to

$$\mathbf{l}^* = (l_G^*, l_I^*, l_T^*, l_{DT}^*)' = (.103, .353, .577, .887)' \tag{70}$$

Thus the equilibrium location on the political line of *all* newspapers *but* the DT shift to the Left. This finding is akin to results in the non-strategic firm setting of Behringer (2007) based on market data only. The implied equilibrium prices in the second stage are

$$\mathbf{p}^{N*}(\mathbf{l}^*) = (p_G^*(\mathbf{l}^*), p_I^*(\mathbf{l}^*), p_T^*(\mathbf{l}^*), p_{DT}^*(\mathbf{l}^*))' = (1.4, 1.158, 1.128, 1.647)' \tag{71}$$

so that *all* equilibrium prices *but* that of the DT go down. The equilibrium market share vector of the readers market is now

$$\mathbf{ms}^N(\mathbf{p}^{N*}(\mathbf{l}^*)) = (.18, .279, .357, .185)' \tag{72}$$

so that market shares of *all* newspapers *but* that of the T go down.

Finally equilibrium profits are now

$$\boldsymbol{\pi}^I(\mathbf{p}^{A*}, \mathbf{p}^{N*}(\mathbf{l}^*)) = (.162, .183, .331, .212)' \tag{73}$$

so that equilibrium profits of the G and the I go down but that of the T and the DT go up. Thus by moving further to the political Right, the DT is able to reduce the competitive pressure (and even increase is profit despite its decreasing market share) that results from the T's unilaterally lower extended costs. The I on the other hand, being an interior firm does not have this option as a move to the political Left will automatically increase the competitive pressure from the G. Hence this move is punished with a substantially lower profit, but also the profit of the G declines.

Welfare costs to consumers in this asymmetric case follow from

$$\begin{aligned} ms_G R &= ms_G - l_G = .18 - .103 = .077 \\ ms_G L &= ms_G - ms_G R = .18 - .077 = .103 \\ ms_I R &= (ms_G + ms_I) - l_I = .106 \\ ms_I L &= ms_I - ms_I R = .279 - .106 = .173 \\ ms_T R &= (ms_G + ms_I + ms_T) - l_T = .239 \\ ms_T L &= ms_T - ms_T R = .118 \\ ms_{DT} R &= 1 - l_{DT} = 1 - .887 = .113 \\ ms_{DT} L &= ms_{DT} - ms_{DT} R = .185 - .113 = .072 \end{aligned} \tag{74}$$

as

$$WC_{AC} = t(\int_0^{0.077} x^2dx + \int_0^{0.103} x^2dx + \int_0^{0.106} x^2dx + \int_0^{0.173} x^2dx + \int_0^{0.239} x^2dx + \int_0^{0.118} x^2dx + \int_0^{0.113} x^2dx + \int_0^{0.072} x^2dx) + \mathbf{p}^{N*}(\mathbf{l}^*)'\mathbf{ms}^N(\mathbf{p}^{N*}(\mathbf{l}^*)) = 8.343 \times 10^{-3}t + 1.2825 \tag{75}$$

again with $t = 10$ $(\alpha = 1)$ this becomes $WC_{AC} = 1.3659$.

Comparing with the symmetric situation where $WC_S = 1.4292$ we note that the cost drop of the times leads to higher price welfare of consumers but the new and more asymmetric locations to slightly higher location welfare costs. Hence whether the cost drop is advantageous to consumers in general will depend on the level cost saving and the level of t.

Calculations for the system where reservation constraints are *all binding* are substantially simpler. With binding constraints, locations decisions (and hence the first stage of the game) does not matter for profit considerations for neither interior nor non-interior firms. A small location change of a paper will lead to a loss of customers on one side that is exactly compensated for by the gain of customers on the other not affecting a neighbours market share. This move will be profit neutral as price choices are (from (61)) dominant strategies. Parameters (and in particular the reservation price R) have to be chosen such that the unit-line constraint to (symmetric) market shares remains satisfied. Given this constraint we then know that equilibrium profits will also be symmetric and decreasing in t and a_i.

5 Conclusion

We propose a theoretical model of the newspaper market with (more than) 4 firms encompassing demand for differentiated products on both sides of the market and profit maximization by competing oligopolistic publishers. These publishers recognize the existence of indirect network effects between the two sides of the market as they choose first the political position, then simultaneously the cover prices and the advertising price

We find that the form of the advertising side as proposed in model of Gabszewicz at. al. (2001, 2002) extends to quadropoly and also that the "full pass through" result of the advertising revenue on equilibrium prices holds. We show that in general the concern for a *Pensée Unique* as a result of advertising financing is increasing. Also, the best guardian against such a Pensée Unique i.e. full minimum differentiation is a partial (n-2) firm minimum differentiation with the remaining two firms covering the extremes of the political spectrum.

For the particular case of *quadropoly* we find that despite the fact that for a large advertising revenue component equilibrium reader prices for the newspapers will fall to zero, unlike in duopoly the tendency to produce overly similar papers is bounded by the fact that another pure strategy Nash Equilibrium exists. The implications for consumer welfare of a drastic but *symmetric* advertising demand increases are thus more comforting.

Eventually we derive the location equilibrium by simulation for an *asymmetric advertising shock* for The Times only to the symmetric situation as motivated by occurences in the alleged newspaper price war in the UK in the 1990s (see Behringer & Filistrucchi (2010b) for details).

Findings reveal surprising equilibrium reactions of The Times' competitors and explain asymmetric prices, circulation, advertising volumes, and locations. From a welfare perspective the equilibrium price drops for all of the papers but the Daily Telegraph and is advantageous to consumers. On the other hand the asymmetry produces a less diverse coverage than under symmetry which benefits the readers with extreme political opinions but, on average, results in higher welfare costs. The net effect of these will thus depend on the size of the production cost savings (advertising increase) relative to the 'transport cost' parameter putting a measure of political diversity in newspapers.

6 Appendix

Proof of Lemma 3:

The inverted system writes as

$$\mathbf{p}^{N*}(\mathbf{l}) = \mathbf{\Omega}^{-1}\frac{1}{2}\mathbf{A}$$

First find the determinant of Ω as

$$\det(\Omega) = \frac{(6l_T - 12l_G)(l_{TD} - l_I) + 6l_T l_{TD} - 3l_T^2 - 3l_I^2}{16(l_T - l_G)(l_{TD} - l_I)} > 0$$

then the inverse $\mathbf{\Omega}^{-1}$ can be written as

$$\begin{vmatrix} -1 & \frac{1}{2} & 0 & 0 \\ \frac{l_T - l_I}{2(l_T - l_G)} & -1 & \frac{l_I - l_G}{2(l_T - l_G)} & 0 \\ 0 & \frac{l_{DT} - l_T}{2(l_{DT} - l_I)} & -1 & \frac{l_T - l_I}{2(l_{DT} - l_I)} \\ 0 & 0 & \frac{1}{2} & -1 \end{vmatrix}^{-1} \equiv \frac{1}{\det(\Omega)} \operatorname{adj}(\Omega) = \frac{1}{\det(\Omega)} \times (-1) \times$$

$$\begin{vmatrix} \frac{3l_T l_{TD} + l_{TD}(l_T - l_I) - 3l_G(l_{TD} - l_I) - 2l_T l_I - l_T^2}{4(l_T - l_G)(l_{TD} - l_I)} & \frac{4l_{TD} - l_T - 3l_I}{8(l_{TD} - l_I)} & \frac{l_I - l_G}{4(l_T - l_G)} & \frac{(l_I - l_G)(l_T - l_I)}{8(l_T - l_G)(l_{TD} - l_I)} \\ \frac{(l_T - l_I)(4l_{TD} - l_T - 3l_I)}{8(l_T - l_G)(l_{TD} - l_I)} & \frac{4l_{TD} - l_T - 3l_I}{4(l_{TD} - l_I)} & \frac{l_I - l_G}{2(l_T - l_G)} & \frac{(l_I - l_G)(l_T - l_I)}{4(l_T - l_G)(l_{TD} - l_I)} \\ \frac{(l_T - l_I)(l_{TD} - l_T)}{4(l_T - l_G)(l_{TD} - l_I)} & \frac{l_{TD} - l_T}{2(l_{TD} - l_I)} & \frac{3l_T + l_I - 4l_G}{4(l_T - l_G)} & \frac{(l_T - l_I)(3l_T + l_I - 4l_G)}{8(l_T - l_G)(l_{TD} - l_I)} \\ \frac{(l_T - l_I)(l_{TD} - l_T)}{8(l_T - l_G)(l_{TD} - l_I)} & \frac{l_{TD} - l_T}{4(l_{TD} - l_I)} & \frac{3l_T + l_I - 4l_G}{8(l_T - l_G)} & \frac{3l_{TD}(l_T - l_G) - l_G l_T + 4l_G l_I - 2l_T l_I - l_I^2}{4(l_T - l_G)(l_{TD} - l_I)} \end{vmatrix}$$

Summing *each of the rows* in the adjoint Matrix $\operatorname{adj}(\Omega)$ yields the same value, namely

$$\sum_{\text{col}} \text{row} \operatorname{adj}(\Omega) = -(\frac{3}{2} - \frac{3\,(l_T - l_I)^2}{8\,(l_T - l_G)\,(l_{TD} - l_I)})$$

and by premultiplying with the inverse of the determinant $\det(\Omega)$ yields

$$\frac{1}{\det(\Omega)} \sum_{\text{col}} \operatorname{adj}(\Omega) = (-1) \times 2 \;\forall l_G, l_I, l_T, l_{DT}$$

With $\mathbf{p}^{N*}(\mathbf{l}) = \mathbf{\Omega}^{-1}\frac{1}{2}\mathbf{A}$ and all rows of the inverse of Ω summing to -2 note that each element of the vector $\mathbf{A}$ is linear in A_i and premultiplied by the scalar $1/2$. For a common advertising demand density $A_i = A \;\forall\; i = G, I, T, DT$ therefore equilibrium price satisfies $\partial p_i^{N*}(\mathbf{l})/\partial A = -1.$■

Proof of Proposition 8:
For an even number of firms n we find equilibrium prices as

$$p_1^N(\mathbf{l}) = \frac{1}{2}c_1^N + \frac{1}{2}(l_2 - l_1)(l_1 + l_2)\frac{t}{\alpha} + \frac{1}{2}p_2^N(\mathbf{l}) - \frac{1}{2}A_1$$

$$p_2^N(\mathbf{l}) = \frac{1}{2}c_2^N + \frac{(l_3 - l_2)(l_2 - l_1)t}{(l_3 - l_1)\alpha}\left(\frac{l_3 - l_1}{2} + \frac{\alpha p_3^N(\mathbf{l})}{2t(l_3 - l_2)} + \frac{\alpha p_1^N(\mathbf{l})}{2t(l_2 - l_1)}\right) - \frac{1}{2}A_2$$

$$p_3^N(\mathbf{l}) = \frac{1}{2}c_3^N + \frac{(l_4 - l_3)(l_3 - l_2)t}{(l_4 - l_2)\alpha}\left(\frac{l_4 - l_2}{2} + \frac{\alpha p_4^N(\mathbf{l})}{2t(l_4 - l_3)} + \frac{\alpha p_2^N(\mathbf{l})}{2t(l_3 - l_2)}\right) - \frac{1}{2}A_3$$

$$\vdots$$

$$\begin{aligned} p_{(n-2)}^N(\mathbf{l}) &= \frac{1}{2}c_{(n-2)}^N + \frac{\left(l_{(n-1)} - l_{(n-2)}\right)\left(l_{(n-2)} - l_{(n-3)}\right)t}{(l_{(n-1)} - l_{(n-3)})\alpha} \times \\ &\left(\frac{l_{(n-1)} - l_{(n-3)}}{2} + \frac{\alpha p_{(n-1)}^N(\mathbf{l})}{2t\left(l_{(n-1)} - l_{(n-2)}\right)} + \frac{\alpha p_{(n-3)}^N(\mathbf{l})}{2t\left(l_{(n-1)} - l_{(n-3)}\right)}\right) - \frac{1}{2}A_{(n-2)} \end{aligned}$$

$$\begin{aligned} p_{(n-1)}^N(\mathbf{l}) &= \frac{1}{2}c_{(n-1)}^N + \frac{\left(l_n - l_{(n-1)}\right)\left(l_{(n-1)} - l_{(n-2)}\right)t}{(l_n - l_{(n-2)})\alpha} \times \\ &\left(\frac{l_n - l_{(n-2)}}{2} + \frac{\alpha p_{(n-1)}^N(\mathbf{l})}{2t\left(l_n - l_{(n-1)}\right)} + \frac{\alpha p_{(n-2)}^N(\mathbf{l})}{2t\left(l_n - l_{(n-2)}\right)}\right) - \frac{1}{2}A_{(n-1)} \end{aligned}$$

$$p_n^N(\mathbf{l}) = \frac{1}{2}c_n^N + l_n(1 - \frac{1}{2}l_n)\frac{t}{\alpha} - l_{(n-1)}(1 - \frac{1}{2}l_{(n-1)})\frac{t}{\alpha} + \frac{1}{2}p_{(n-1)}^N(\mathbf{l}) - \frac{1}{2}A_n$$

Adding up the location terms analogous to (56) and using symmetry $l_n = (1 - l_1)$, $l_{n-1} = (1 - l_2)$... we can derive the condition (56) for n firms as

$$\left(\sum_{i=1}^{n}(A_i - c_i)\right) = (n(A - c)) > 2 * t\Psi \equiv$$
$$2 * t\left(\begin{array}{c} (l_2 - l_1)(l_1 + l_2) + (l_3 - l_2)(l_2 - l_1) + (l_4 - l_3)(l_3 - l_2) + ... \\ + \left(l_{\frac{n}{2}-1} - l_{\frac{n}{2}-2}\right)\left(l_{\frac{n}{2}-2} - l_{\frac{n}{2}-3}\right) + \left(l_{\frac{n}{2}} - l_{\frac{n}{2}-1}\right)\left(l_{\frac{n}{2}-1} - l_{\frac{n}{2}-2}\right) + \left(l_{\frac{n}{2}+1} - l_{\frac{n}{2}}\right)\left(l_{\frac{n}{2}} - l_{\frac{n}{2}-1}\right) \end{array}\right)$$

where we have added symmetry in advertising revenue and demands. Also by symmetry we have

$$l_{\frac{n}{2}+1} = 1 - l_{\frac{n}{2}}$$

so the last line reads

$$...+\left(l_{\frac{n}{2}-1}-l_{\frac{n}{2}-2}\right)\left(l_{\frac{n}{2}-2}-l_{\frac{n}{2}-3}\right)+\left(l_{\frac{n}{2}}-l_{\frac{n}{2}-1}\right)\left(l_{\frac{n}{2}-1}-l_{\frac{n}{2}-2}\right)+\left(\left(1-l_{\frac{n}{2}}\right)-l_{\frac{n}{2}}\right)\left(l_{\frac{n}{2}}-l_{\frac{n}{2}-1}\right)$$

as n is even and firms symmetric around $1/2$. Maximizing Ψ subject to the ordering we find the first order conditions

$$\begin{aligned}
\frac{\partial\Psi}{\partial l_1} &= -2l_1+l_2-l_3=0\\
\frac{\partial\Psi}{\partial l_2} &= l_1+2l_3-l_4=0\\
\frac{\partial\Psi}{\partial l_{j\geq 3}} &= -l_{j-2}+2l_{j-1}-2l_j+2l_{j+1}-l_{j+2}=0\\
&\quad ...\\
\frac{\partial\Psi}{\partial l_{\frac{n}{2}-2}} &= -l_{\frac{n}{2}-4}+2l_{\frac{n}{2}-3}-2l_{\frac{n}{2}-2}+2l_{\frac{n}{2}-1}-l_{\frac{n}{2}}=0\\
\frac{\partial\Psi}{\partial l_{\frac{n}{2}-1}} &= l_{\frac{n}{2}-2}+3l_{\frac{n}{2}}-1=0\\
\frac{\partial\Psi}{\partial l_{\frac{n}{2}}} &= -l_{\frac{n}{2}-2}+3l_{\frac{n}{2}-1}-4l_{\frac{n}{2}}+1=0
\end{aligned}$$

Note that all of these but $\partial\Psi/\partial l_{\frac{n}{2}-1}$ are strictly concave. If $l_{\frac{n}{2}-1}=0$ we have from $\partial\Psi/\partial l_{\frac{n}{2}-2}$ that

$$l_{\frac{n}{2}-3}=\frac{1}{2}l_{\frac{n}{2}-4}+2l_{\frac{n}{2}-2}+\frac{1}{2}l_{\frac{n}{2}}$$

which contradicts the ordering. If $l_{\frac{n}{2}-1}=\frac{1}{2}$ the ordering implies that $l_{\frac{n}{2}}=\frac{1}{2}$ too. Now we have that

$$l_{\frac{n}{2}}=1-l_{\frac{n}{2}-1}$$

and the last part of Ψ cancels. The remainder is now

$$...+\left(l_{\frac{n}{2}-1}-l_{\frac{n}{2}-2}\right)\left(l_{\frac{n}{2}-2}-l_{\frac{n}{2}-3}\right)+\left(\left(1-l_{\frac{n}{2}-1}\right)-l_{\frac{n}{2}-1}\right)\left(l_{\frac{n}{2}-1}-l_{\frac{n}{2}-2}\right)$$

so that now $\partial\Psi/\partial l_{\frac{n}{2}-2}$ is no longer concave. The entire problem unravels from the back and we eventually find that $\partial\Psi/\partial l_1$ implies that

$$l_1=0$$

The optimal value that the condition above takes at these locations is

$$A-c>\frac{1}{2n}t$$

for any $n\geq 4$. The proof for a general odd number of firms is similar.■

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