Public Good Provision with many Agents: An Example

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Abstract

This paper provides a simple example of a general mechanism that solves the free riding problem when many agents are both users and beneficiaries of the public good such as in the case of open source software or social networks.

1 Introduction

Investigations of the provision of public goods have a long tradition in the theory of public finance. Such public goods are known to raise the problem of "free riding" at least since Samuelson (1954) and it can easily be shown that even in full information context the problem becomes more severe when there are many agents involved.

Accepting the fact that there usually is asymmetric information and the provider of the public good (usually assumed to be the government) will often not know in advance how people value the public good individually (and thus in aggregate) the optimal decision about provision (or sometimes about the optimal provision level) is also not known ex-ante. The literature of *mechanism design* has taken this information asymmetry very seriously and tried to offer support to policy makers regarding these issues by designing a device that intends to elicit these true individual preferences, that is to satisfy *incentive compatibility* constraints (for a very recent introduction to these issues with much emphasis on the public good provision problem see Börgers, (2010)).

Much of the early theoretical work has been conducted on the basis of the proposal made by Groves (1973), often called the "Clarke-Groves Mechanism" which allows for implementation in *dominant strategies*, i.e. optimal own announcements about valuations will not depend on what other agents do or about expectations about their true valuations.

This form of implementation is also used in Groves' & Ledyard's, (1977) proposed solution to the free rider problem. Important contributions in this area are summarized in the work of Green & Laffont, (1979).

A rising concern of theorists for the need to advocate mechanisms to the policy maker that are neutral on the budget, or *budget-balanced*, i.e. where the incentive scheme underlying the mechanism does not require extra governmental funding, i.e. money that should rather go to the provision of the public good itself, has led them to look for a weaker form of implementation, namely *Bayesian implementation*. Here one's optimal strategy will depend on the own valuation and information about the other's type (usually the statistical properties of some parent distribution that is assumed to be commonly known). Hence the restriction this implies for the form that a mechanism may take are strict. The important proposal in this direction is by d'Aspremont & Gerard-Varet (1979) and by Arrow, 1979 but in this mechanism another attractive property fails, namely agents may prefer to opt out of the mechanism altogether, i.e. Bayesian *individual rationality* is violated.

Operating within the environment of Bayesian implementation there have been two recent tendencies that have attracted particular interest. First there has been an investigation of mechanisms for *more specific cases of public goods* with non-rival properties , e.g. Baliga & Maskin (2003) investigate mechanisms for the environment.

Secondly there has been an interest in the performance of Bayesian mechanisms in *large economies* i.e. what one may be able to say about the performance of a mechanism once the number of agents for which the public good is to be provided gets very large. Early fundamental results such as those by Mailath & Postlewaite, (1990) have been predominantly negative. They conclude that provision of the public good will go to zero as the number of agents increases. The underlying intuition for this result is that, unlike in a private good setting where efficiency can usually be obtained in the limit (e.g. Cripps & Swinkels (2006)), with many agents the probability of being the agent who by his truthful announcement of valuation '*tilts*' the balance between non-provision and provision in favour of the latter will be strictly decreasing. As this possibility of being pivotal for the outcome of the provision effort is commensurable with the amount that an individual is willing to voluntarily contribute for the good which comes at a constant per-capita cost (e.g. national defense) the former effect will be dominating with sufficiently many agents. This leads to non-provision in the limit.

[0] Sometimes this proposal is even called the "Vickery-Clarke-Groves Mechanism" to empasize that transfers resemble the payments in a second price (Vickery) auction.

For the open source context this translates as follows: If there are sufficiently many agents I may just wait for yet another module of the software (e.g. a diver for my own printer) to be developed even if I may quite easily write this piece of source code myself. In other words the probability of being pivotal decreases as there are many agents so that following the findings of Mailath and Postlewaite (1990) we should expect the development process to be slow when there is a myriad of agents involved.

Hellwig, (2003) has also looked at the issue for both an a priori bounded and an unbounded public good and finds that assuming a fixed cost for the public good (e.g. a lighthouse) is sufficient for the negative limit result of Mailath & Postlewaite to be reversed. More precisely he is able to show that second best provision levels converge in distribution to first best, (full information) levels. For the unbounded case he is able to show that the difference between second best and first best levels becomes large and hence there will be serious underprovision of the public good with many agents. In the latter analysis Hellwig has to make a specific assumption about the form that the cost function of the public good is supposed to have.

Recent work of Behringer, (2011) contributes to both these tendencies. The work is motivated by a dissatisfaction with the sensitivity of the limit results with respect to the ex-ante assumption about the cost technology and also by the observation of rapid development of *public information goods* such as Open Source Software (despite there being a myriad of agents involved). The author is able to endogenize the choice of the cost technology at the expense of having a more specific provision technology.

This *direct provision technology* has the particular feature that it assumes that the agents that traditionally only consume the public good are the very same agents who may or may not provide it. There is thus no need for government provision as in the 'standard' public good provision context. This form of provision seems to dominate important recent development in information technologies like the rapid developments of multi-lingual online encyclopaedia such as Wikipedia or complex networks such as Myspace or Facebook.

In particular the above mentioned case of Open Source Software such as Linux (see Raymond, (1996) for fascinating details of this movement and the homepage of the MIT Free/Open Source Research Community containing virtually hundreds of articles that address these issue form multidisciplinary points of view) has intrigued economists for quite some time now. Some have even argued it to be some 'third way' of production that efficiently aggregates many voluntary contributors in one production process. Others such as Lerner & Tirole, (1994) have argued that rather 'delayed rewards' to specific programmers may be responsible for their willingness to contribute. The occupation with cases of such "social production" excites disciples well beyond the focus of economists i.e. Benkler (2002) for a legal approach.

2 The (2011) Model

There are n agents with private valuation parameter (type) $\theta_i \in \Theta, i = 1, 2...n$, $\Theta \equiv [\underline{\theta}, \bar{\theta}]$ which are realizations of independently and identically distributed (i.i.d.) random variables $\tilde{\theta}_i$. The random variables $\tilde{\theta}_i$ are drawn from identical distributions that have a continuous and strictly positive density $f(\theta_i)$, and a cumulative distribution function $F(\theta_i)$.

The provision of the public good is an all or nothing decision, i.e. the level of the public good provision is fixed and will take place with some probability $r(\boldsymbol{\theta}) : \Theta^n \to [0, 1]$ where the total vector of types is denoted by $\boldsymbol{\theta} \in [\underline{\theta}, \bar{\theta}]^n$. Employing the revelation principle we can restrict ourselves to incentive compatible direct mechanisms.

A mechanism (allocation) is a triple of functions $\langle r(\boldsymbol{\theta}), \mathbf{p}(\boldsymbol{\theta}), \mathbf{z}(\boldsymbol{\theta})\rangle$ where $\mathbf{p}(\boldsymbol{\theta})$ is the vector of total effort or contributions to the public good with generic elements $(p_i(\boldsymbol{\theta})) : \Theta^n \to [0, 1]$ and $\mathbf{z}(\boldsymbol{\theta})$ is a vector of net side payments with generic elements $(z_i(\boldsymbol{\theta})) : \Theta^n \to \Re$.

Agent's net utilities will then consist of the real allocations $r(\boldsymbol{\theta})$ and $\mathbf{p}(\boldsymbol{\theta})$ and the allocation of net side payments $\mathbf{z}(\boldsymbol{\theta})$. Ex-post utilities are denoted as

$$u_i = \theta_i r(\boldsymbol{\theta}) - p_i(\boldsymbol{\theta}) - z_i(\boldsymbol{\theta}) \tag{1}$$

The above is a standard setup for a Bayesian public good mechanism design problem and can be found in standard texts on the topic such as Börgers (2008) or Jackson (2003). What is new to this setup is the idea that for certain public goods that may be summarized under the heading "*public information goods*" the sets of agents who benefit from the good and the set of agents who provide it (usually thought of as a government) are not disjoint. Taking seriously this dichotomy of agents allows to set up two separate contracts that the agent may sign, one with their "user hat" on and the other one as a "provider".

As the agent is a user as well as a direct provider of the public good the utility function can be decomposed into two parts as

$$u_i = \underbrace{\theta_i r(\boldsymbol{\theta}) - z_{Ui}(\boldsymbol{\theta})}_{user} \underbrace{- p_i(\boldsymbol{\theta}) + z_{Pi}(\boldsymbol{\theta})}_{provider} \tag{2}$$

respectively, so that net side payments are defined as

$$z_i(\boldsymbol{\theta}) \equiv z_{Ui}(\boldsymbol{\theta}) - z_{Pi}(\boldsymbol{\theta}) \tag{3}$$

The other novel ingredient into the standard public good mechanism design setting is the *direct provision technology* which implies that each agent i can provide the good with some probability (effort) $p_i(\boldsymbol{\theta})$ directly. The total probability of the good being provided independently with n agents is thus, by independence,

$$r(\boldsymbol{\theta}) = 1 - \Pi_{i=1}^n (1 - p_i(\boldsymbol{\theta})) \tag{4}$$

An allocation is *feasible* if side payments satisfy

$$\sum_{i=1}^{n} z_i(\boldsymbol{\theta}) \equiv \sum_{i=1}^{n} (z_{Ui}(\boldsymbol{\theta}) - z_{Pi}(\boldsymbol{\theta})) \;\; \geq 0 \; \forall \; \boldsymbol{\theta} \tag{5}$$

We also make an *implementation assumption*

$$p_1(\boldsymbol{\theta}) = ... = p_n(\boldsymbol{\theta}) = p(\boldsymbol{\theta}) = z_{Pi}(\boldsymbol{\theta}) \; \forall \; \boldsymbol{\theta} \tag{6}$$

i.e. the technology that maps individual efforts into the total provision probability as in (4) is fully symmetric and which is a natural choice because we do not model the fact that agents may also have different abilities for providing the public good. Total cost of provision is then simply $k_n(r(\boldsymbol{\theta})) = np(\boldsymbol{\theta})$ .This assumption clearly implies that no agent will ever regret to have taken part in the provision process, independently of the provision outcome.

We now denote each agents interim probability of provision estimate as

$$\rho_i(\theta_i) \equiv \int r(\theta_i, \boldsymbol{\theta}_{-i}) dF^{n-1}(\boldsymbol{\theta}_{-i}) \tag{7}$$

Aggregating, amalgamating, and adequately transforming the individual constraints on the mechanism, for the total contract then the following well

known result is obtained

Lemma 1 *For any probability of provision $r(\boldsymbol{\theta})$ such that $\rho_i(\theta_i)$ is non-decreasing in θ_i, there exist net side payments $\mathbf{z}(\boldsymbol{\theta})$ such that $< r(\boldsymbol{\theta}), \mathbf{z}(\boldsymbol{\theta}) >$ is interim incentive compatible, interim individual rational, and weakly feasible iff*

$$\int \left(\sum_{i=1}^{n} \left(\theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)} \right) r(\boldsymbol{\theta}) - k_n(r(\boldsymbol{\theta})) \right) dF^n(\boldsymbol{\theta}) \geq 0 \tag{8}$$

Proof: See Behringer (2008).■

It has been shown recently that the step from dominant strategy implementation and the implied non-balancedness to Bayesian implementation with its requirement of Bayesian budget balancedness. (or weak feasibility here) is not too bothersome even if, as a policy maker, we should be concerned with ex-post budget balancedness. This is because once we have found a mechanism that indeed satisfies incentive compatibility, individual rationality and budget balance at the individually expected level an alternative mechanism can be constructed that satisfies ex-post budget balance (see Börgers & Norman, 2008).

The mechanism designer has to solve the following program for the weakly feasible overall mechanism that implements the contracts under interim individual rationality and incentive compatibility and generates non-negative expected social benefit. This program is: Maximize expected social welfare programme $\mathcal{P}$

$$Max_{r(\boldsymbol{\theta})} \left\{ \int \left(\sum_{i=1}^{n} \theta_i r(\boldsymbol{\theta}) - k_n(r(\boldsymbol{\theta})) \right) dF^n(\boldsymbol{\theta}), 0 \right\} \tag{9}$$

$$s.t.\ \rho_i(\theta_i) \text{ non-decreasing in } \theta_i \text{ and} \tag{10}$$

$$s.t. \quad \int \left(\sum_{i=1}^{n} \left(\theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)} \right) r(\boldsymbol{\theta}) - k_n(r(\boldsymbol{\theta})) \right) dF^n(\boldsymbol{\theta}) \geq 0 \tag{11}$$

i.e. subject to *monotonicity*, which can be guaranteed by assuming that the distribution satisfies the *hazard rate condition*, i.e. that the "virtual utility" $\theta_i - (1 - F(\theta_i))/f(\theta_i)$ is non-decreasing in θ_i.

The *ex-post efficient* (or **first best**) provision rule given by the first order condition for (9) equates marginal cost and marginal benefit (here equal to total benefit as we are concerned with a good that is non-rival in consumption) and thus

$$\frac{\partial k_n(r(\boldsymbol{\theta}))}{\partial r} = (1 - r)^{\frac{1}{n} - 1} = \sum_{i=1}^{n} \theta_i \tag{12}$$

The first best provision rule is thus

$$r^{FB*}(\boldsymbol{\theta}) = \begin{cases} 1 - (\sum_{i=1}^{n} \theta_i)^{-\frac{n}{n-1}} & \text{if } \sum_{i=1}^{n} \theta_i > 1 \\ 0 \text{ otherwise} \end{cases} \tag{13}$$

This program, using a Lagrange multiplier approach programme $\mathcal{P}$ can be rearranged as

$$Max_{r(\boldsymbol{\theta}),\lambda} \left\{ \begin{array}{c} (1 + \lambda) \left(\int \left(\sum_{i=1}^{n} \theta_i r(\boldsymbol{\theta}) - k_n(r(\boldsymbol{\theta})) \right) dF^n(\boldsymbol{\theta}) \right) \\ -\lambda \int \left(\sum_{i=1}^{n} \left(\frac{1 - F(\theta_i)}{f(\theta_i)} \right) r(\boldsymbol{\theta}) \right) dF^n(\boldsymbol{\theta}), 0 \end{array} \right\} \tag{14}$$

so that the **second best** provision rule can be written as (see Lemma 4 in Behringer, 2008)

$$r^{SB*}(\boldsymbol{\theta}) = \begin{cases} (1 - (\sum_{i=1}^{n} \theta_i)^{-\frac{n}{n-1}}) \text{ if } (1 + \lambda)(\sum_{i=1}^{n} \theta_i \times (1 - (\sum_{i=1}^{n} \theta_i)^{-\frac{n}{n-1}}) - k_n(r(\boldsymbol{\theta}))) > \\ \lambda \sum (\frac{1 - F(\theta_i)}{f(\theta_i)})(1 - (\sum_{i=1}^{n} \theta_i)^{-\frac{n}{n-1}}) \\ 0 \text{ otherwise} \end{cases}$$

The original paper now goes on to investigate the statistical *large sample properties* of the mechanism and finds the following comforting result. Given the number of agents within the mechanism is sufficiently large so that the first best provision level almost surely converges to provision with certainty (which can be easily seen by looking at 13), the second best provision levels also converge in probability to first best, (full information) levels. Hence the early negative predictions for public good provision in large societies by Mailath & Postlewaite (1990) can be overtured *without* making the restrictive assumption that the total cost of provision is independent of the size of the population which is to benefit from it (see Behringer, 2008, Theorem).

Instead of looking at such large sample properties we will investigate a very simple example in order to compare how the mechanism performs relative to other more standard mechanisms in small samples.

3 Example with the direct Provision Technology

For the special case that there are $n = 2$ agents with private valuation parameter (type) $\theta_i \in \Theta, i = 1, 2,\ \Theta \equiv [0, 1]$ where the random variables $\tilde{\theta}_i$ are drawn from identical *uniform* distributions that have a continuous and strictly positive density $f(\theta_i) = 1$, and a cumulative distribution function $F(\theta_i) = \theta_i$. Hence the joint distribution of valuations is from $\Theta^{n=2} \equiv [0, 1]^2$, the unit square.

Clearly the hazard rate condition holds given the uniform distribution as $\theta_i - (1 - F(\theta_i))/f(\theta_i) = 2\theta_i - 1$ is non-decreasing in θ_i.

Given the direct provision technology, total costs are $k_n(r(\boldsymbol{\theta})) = 2p(\boldsymbol{\theta})$ and given the independence assumption (4) this reduces to

$$k(r(\boldsymbol{\theta})) = 2 - 2\sqrt{(1 - r(\boldsymbol{\theta}))} \in [0, 2]$$

which is a *strictly convex function* over the domain of $r(\boldsymbol{\theta}) \in [0, 1)$.

From above the first best provision rule reduces to

$$r^{FB*}(\boldsymbol{\theta}) = \begin{cases} 1 - (\theta_1 + \theta_2)^{-2} \text{ if } \theta_1 + \theta_2 > 1 \\ 0 \text{ otherwise} \end{cases} \tag{15}$$

so this excludes the case that one agent alone can have a sufficiently high valuation to justify a provision effort.

Also from above the second best provision rule can be rewritten as

$$r^{SB*} = \begin{cases} (1 - (\sum_{i=1}^n \theta_i)^{-\frac{n}{n-1}}) \text{ if } \sum_{i=1}^n \theta_i > n\frac{(\sum_{i=1}^n \theta_i) - (\sum_{i=1}^n \theta_i)^{\frac{n}{n-1}}}{1 - (\sum_{i=1}^n \theta_i)^{\frac{n}{n-1}}} + \frac{\lambda}{1+\lambda}\sum(\frac{1-F(\theta_i)}{f(\theta_i)}) \\ 0 \text{ otherwise} \end{cases} \tag{16}$$

We find the following result: Given uniform distributions and two agents

Lemma 2 *With the direct Provision Technology as in Behringer (2008),* $k(r(\boldsymbol{\theta})) = 2-2\sqrt{(1-r(\boldsymbol{\theta}))} \in [0,2]$, $f(\theta)=1$, *and* $n=2$, *the utilitarian designer will chose a mechanism with the allocation rule*

$$r^* = \begin{cases} (1-(\theta_1+\theta_2)^{-2} \text{ if } \theta_1+\theta_2 \gtrsim s^+ \\ 0 \text{ otherwise} \end{cases} \tag{17}$$

where $s^+ \approx 1.4$ *is the unique solution in* $s \in [1, \frac{1}{2}(1+\sqrt{5})]$ *to*

$$s^4 - 6s^3 + 12s^2 - 3(\ln 2)\,s - 11s + 3(\ln s)\,s + 6 = 0 \tag{18}$$

As first best ($\lambda = 0$) produces if $\theta_1+\theta_2 > 1$ and second best only produces if $\theta_1+\theta_2 > 1.4$ we have a higher threshold which implies that for this example with $n=2$ second best is bounded from first best.

Proof:
From above (16) reduces to

$$r^{SB*} = \begin{cases} (1-(\theta_1+\theta_2)^{-2}) \text{ if } (1+\lambda)((\theta_1+\theta_2)\times(1-(\theta_1+\theta_2)^{-2})-k) > \\ \lambda(2-\theta_1-\theta_2)(1-(\theta_1+\theta_2)^{-2}) \\ 0 \text{ otherwise} \end{cases} \tag{19}$$

which can be rewritten as

$$\begin{aligned} r^{SB*} &= (1-(\theta_1+\theta_2)^{-2}) \text{ if } \theta_1+\theta_2 > \\ & 2\frac{(\theta_1+\theta_2)-(\theta_1+\theta_2)^2}{1-(\theta_1+\theta_2)^2} + \frac{\lambda}{1+\lambda}(2-\theta_1-\theta_2) \end{aligned} \tag{20}$$

The quadratic on the RHS of the inequality constraint has two roots

$$\theta_1+\theta_2 = \frac{1}{2(1+2\lambda)}\left(1+2\lambda \pm \sqrt{(20\lambda^2+12\lambda+1)}\right)$$

and so the inequality will hold if $\theta_1+\theta_2$ is either "large" or "small". We take the larger root as for any $\lambda > 0$ the lower root is $\in (\frac{1}{2}-\frac{1}{2}\sqrt{5}, 0) = (-0.618, 0)$ which cannot be the case as $\theta_1+\theta_2 \geq 0$ and the larger root is $\in (1, \frac{1}{2}+\frac{1}{2}\sqrt{5}) = (1, 1.618)$.
Thus we find that

$$r^{SB*} = \begin{cases} (1-(\theta_1+\theta_2)^{-2}) \text{ if } \theta_1+\theta_2 > \underbrace{\frac{1}{2(1+2\lambda)}\left(2\lambda+1+\sqrt{(20\lambda^2+12\lambda+1)}\right)}_{s} \\ 0 \text{ otherwise} \end{cases} \tag{21}$$

holds.

The RHS of the condition is denoted by s and we seek to find the optimal value for s. What is the bound on s? For $\lambda = 0$ we find $s = 1$ the first best outcome and

$$\lim_{\lambda \to \infty} (s) = \frac{1}{2}(1 + \sqrt{5}) \approx 1.618 \tag{22}$$

Also s is monotone increasing in λ and hence we need only be concerned with $s \in [1, \frac{1}{2}(1+\sqrt{5})]$, i.e. with the case $s > 1$ here.

From above, second best costs are

$$k(r^{SB*}(\boldsymbol{\theta})) = 2 - 2\sqrt{(1 - r^{SB*}(\boldsymbol{\theta}))} = 2\frac{\theta_1 + \theta_2 - 1}{\theta_1 + \theta_2}$$

My costs of $k = np = 2 - 2\sqrt{(1-r)}$ with $r = (1 - (\theta_1 + \theta_2)^{-2})$ then costs are

$$k = 2 - 2\sqrt{(1 - (1 - (\theta_1 + \theta_2)^{-2}))} = 2\frac{\theta_1 + \theta_2 - 1}{\theta_1 + \theta_2} \tag{23}$$

As $s > 1$ a single agent cannot have a valuation that satisfies $\theta_1 + \theta_2 > s$ hence

$$\begin{aligned} \Pr\{\theta_1 + \theta_2 > s | \, s > 1\} &= \Pr\left\{\theta_1 + \theta_2 > s | \, \theta_1 + \max \bar{\theta}_2 > s, \theta_2 + \bar{\theta}_1 > s\right\} = \\ & \quad \Pr\{\theta_1 + \theta_2 > s | \, \theta_1 > s - 1, \theta_2 > s - \theta_1\} \end{aligned}$$

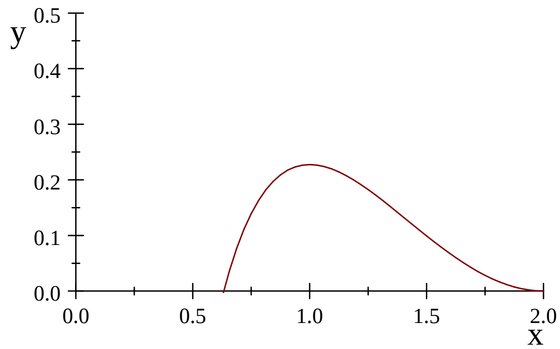
Expected costs are thus

$$C(s) = \int_{s-1}^{1} \int_{s-\theta_1}^{1} k(\theta_1 + \theta_2)) f(\theta) d\theta_2 d\theta_1 \tag{24}$$

or

$$C(s) = \int_{s-1}^{1} \int_{s-\theta_1}^{1} (2\frac{\theta_1 + \theta_2 - 1}{\theta_1 + \theta_2}) d\theta_2 d\theta_1 = 8 - 4\ln 2 - 6s + 4\ln s + s^2 \tag{25}$$

and a plot reveals



that expected costs are strictly decreasing in the relevant range for s. Hence a λ implies a higher cutoff s and a lower expected cost.

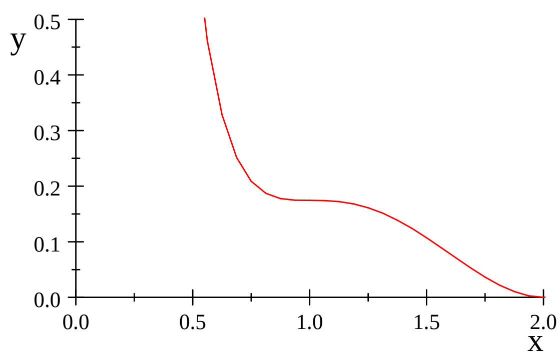
Expected payments of one agent are thus

$$\int_{s-1}^{1}\int_{s-\theta_1}^{1} r(\theta_1,\theta_2)(\theta_i - \frac{1-F_i(\theta_i)}{f_i(\theta_i)})f(\theta)d\theta_2 d\theta_1 \tag{26}$$

or

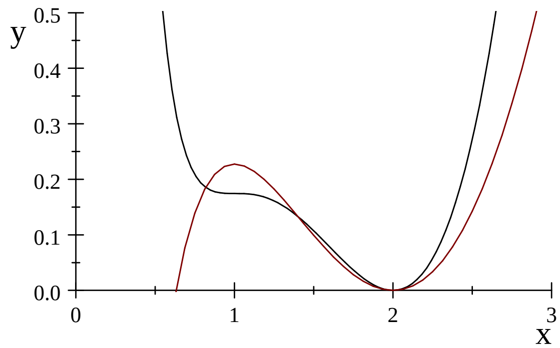
$$\int_{s-1}^{1}\int_{s-\theta_1}^{1} ((1-(\theta_1+\theta_2)^{-2})(2\theta_1-1))d\theta_2 d\theta_1 = \tag{27}$$
$$-\frac{1}{6}\frac{-2s+18(\ln 2)s-6s^2+9s^3-2s^4-18(\ln s)s-12}{s}$$

Total expected payments are twice this, hence



Total expected payments are also strictly decreasing in s given that *both* agents have to provide and hence a higher λ implies a lower total expected payment as expected.

As a necessary condition for an optimum s is that it must be such that ex ante expected expenditure equals ex ante expected costs we look at this case graphically.



A higher constraint multiplier λ (leading to a higher s threshold) will thus decrease both expected revenue.

Alternatively, the difference function (twice the expected payments minus cost) is

$$D(s) = \frac{2}{3}\frac{-11s - 3(\ln 2)s + 12s^2 - 6s^3 + s^4 + 3(\ln s)s + 6}{s} \tag{28}$$

As we are sure that $s \neq 0$ the roots of this difference function are the same as in

$$D(s) = s^4 - 6s^3 + 12s^2 - 3(\ln 2)s - 11s + 3(\ln s)s + 6 = 0 \tag{29}$$

A numerical simulation reveals the critical root to be approximately

$$D(s^+ \approx 1.4) = 0 \tag{30}$$

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Note that there is a second intersection (leaving aside $s = 2$) but that is outside the relevant interval for s.

4 Example with a standard Technology

Following the example in Börgers, 2008, p.59 for the standard technology we find the following result:

Lemma 3 *With a standard Provision Technology such as in Mailath & Postlewaite (1990) or Hellwig (2003), $k = \frac{1}{2}n$ or $k = 1$, $f(\theta) = 1$, and $n = 2$, the utilitarian designer will chose a mechanism with the allocation rule*

$$r^* = \begin{cases} 1 \textit{ if } \theta_1 + \theta_2 \gtrsim s^{++} \\ 0 \textit{ otherwise} \end{cases} \tag{31}$$

where $s^{++} \approx 1.25$.

Proof:

For the standard Provision Technology the first best provision rule at cost $k = 1$ is

$$r^{FB*}(\boldsymbol{\theta}) = \begin{cases} 1 \text{ if } \theta_1 + \theta_2 > 1 \\ 0 \text{ otherwise} \end{cases} \tag{32}$$

which is the same condition as with the direct Provision Technology. The second best condition is hence

$$r^{SB*}(\boldsymbol{\theta}) = \begin{cases} 1 \text{ if } \theta_1 + \theta_2 > 1 + \sum(\frac{1-F(\theta_i)}{f(\theta_i)}) \\ 0 \text{ otherwise} \end{cases}$$

which reduces to

$$r^{SB*}(\boldsymbol{\theta}) = \begin{cases} 1 \text{ if } \theta_1 + \theta_2 > \underbrace{\frac{1+\lambda}{1+2\lambda} + \frac{\lambda}{1+2\lambda}2}_{s} \\ 0 \text{ otherwise} \end{cases}$$

Again we denote the RHS of the inequality by s and note that $s \in [1, 1.5]$.

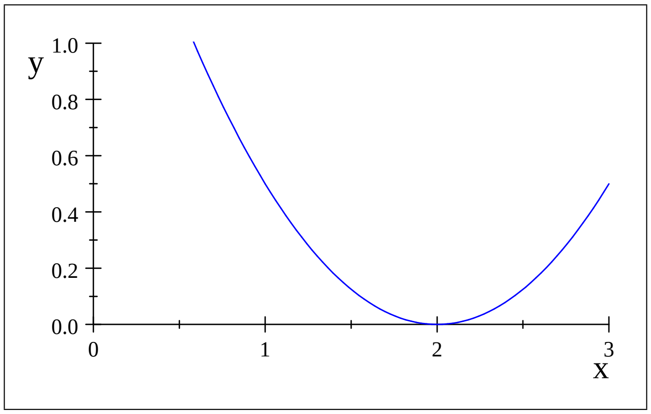
Expected costs are

$$C(s) = \int_{s-1}^{1} \int_{s-\theta_1}^{1} k(\theta_1 + \theta_2))f(\theta)d\theta_2 d\theta_1$$

reduce to

$$C(s) = \int_{s-1}^{1} \int_{s-\theta_1}^{1} 1 d\theta_2 d\theta_1 = \frac{1}{2}(2-s)^2$$

and a plot reveals



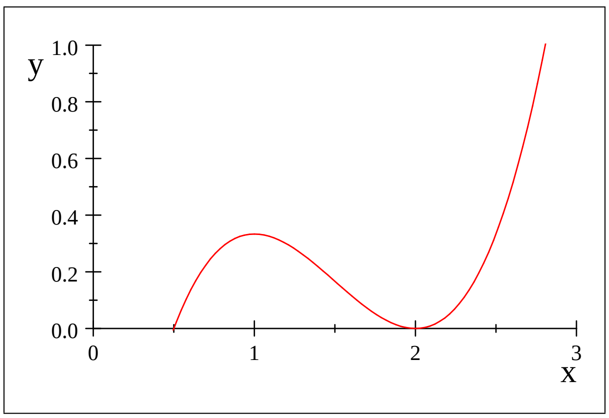
Expected payments are

$$\int_{s-1}^{1}\int_{s-\theta_1}^{1} r(\theta_1,\theta_2)(\theta_i - \frac{1-F_i(\theta_i)}{f_i(\theta_i)})f(\theta)d\theta_2 d\theta_1 \tag{33}$$

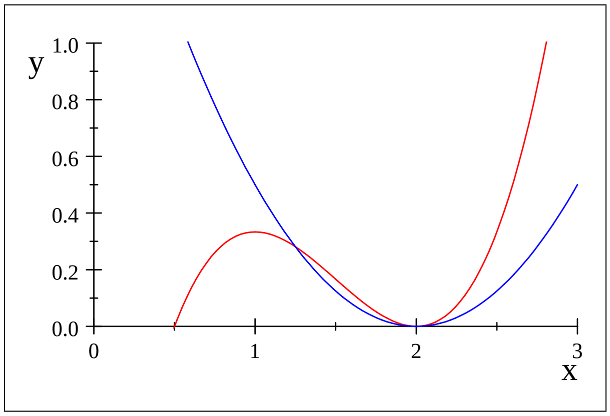
or

$$\begin{aligned}\int_{s-1}^{1}\int_{s-\theta_1}^{1} r(\theta_1,\theta_2)(\theta_i - \frac{1-F_i(\theta_i)}{f_i(\theta_i)})f(\theta)d\theta_2 d\theta_1 &= \int_{s-1}^{1}\int_{s-\theta_1}^{1}(2\theta_1-1)d\theta_2 d\theta_1 = \\ \frac{1}{6}\left(-1+2s\right)\left(s-2\right)^2 &= \frac{1}{6}-\frac{1}{2}(s-1)^2+\frac{1}{3}(s-1)^3\end{aligned}$$

Total expected payments are twice this hence:



As a necessary condition for an optimum s is that it must be such that ex ante expected expenditure equals ex ante expected costs we look at this case graphically.



where the relevant range is $s \in [1, 1.5]$ where we have a unique intersection at $s = 1.25$.

This can also be determined analytically by solving the difference function

$$\begin{aligned} D(s) &= 2(\frac{1}{6} - \frac{1}{2}(s-1)^2 + \frac{1}{3}(s-1)^3) - \frac{1}{2}(2-s)^2 \\ &= \frac{2}{3}s^3 - \frac{7}{2}s^2 + 6s - \frac{10}{3} = 0 \end{aligned}$$

which yields $s_{01} = 1.25$ and $s_{02} = 2$.■

Note that there is a unique intersection here (leaving aside $s = 2$).

However in the relevant range the graphs are qualitatively identical with cost cutting total expected payments from above and then both being tangential at $s = 2$. The important difference to the direct Provision technology is that for lower values of s than the lowest admissible value in *both cases*, i.e. $s = 1$ the standard technology has diverging costs and payments and the direct provision technology has converging costs and payments.

5 Conclusion

From the above examples for the performance of different standard Provision Technologies and that of the direct Provision Technology we can conclude that despite the latter's attractive properties in *large* samples it may perform worse in *small* ones.

6 References

Arrow, K. (1979): "The Property Rights Doctrine and Demand Revelation under Incomplete Information", in Boskin, M. ed. *Economies and Human Welfare*, New York: Academic Press.

Baliga, S. & Maskin, E. (2003): "Mechanism design for the environment", in: Mäler, K.G. & Vincent J. R. (ed.), *Handbook of Environmental Economics*, Edition 1, vol. 1, ch. 7, pages 305-324 Elsevier.

Benkler, Y. (2002): "Coase's Penguin, or Linux and the Nature of the Firm",

112, *Yale Law Journal*, 369.

Behringer, S. (2011): "The Provision of a Public Good with a direct Provision Technology and a Large Number of Agents", mimeo, Universität Bonn, available at http://stefanbehringer.com/.

Börgers, T. (2010): "An Introduction to the Theory of Mechanism Design", mimeo. University of Michigan.

Börgers, T. & Norman, P. (2009): "A Note on Budget Balance under Interim Participation Constraints: The Case of Independent Types" *Economic Theory*, 39, p.477-489.

Cripps, M.W. & Swinkels, J.M. (2006): "Efficiency of Large Double Auctions," *Econometrica*, vol. 74(1), p. 47-92.

d'Aspremont, C. & Gerard-Varet, L.A. (1979): "Incentives and Incomplete Information", *Journal of Public Economics*, 11, p.25-45.

Green, J. & Laffont, J.J. (1979): *Incentives in Public Decision Making*, Amsterdam: North-Holland.

Groves, T. (1973): "Incentives in Teams", *Econometrica*, 41, pp. 617-631.

Groves, T. & Ledyard, J. (1977): "Optimal Allocation of Public Goods: A Solution to the "Free Rider" Problem", *Econometrica*, 45, no. 4, p.783-809.

Hellwig, M.F. (2003): "Public-Good Provision with Many Participants", *Review of Economic Studies*, 70, p.589-614.

Jackson, M. (2003): "Mechanism Theory" in Derigs, U. (ed.) *Encyclopedia of Life Support Systems*, EOLSS Publishers: Oxford UK.

Lerner, J. & Tirole, J. (2002): "Some Simple Economics of Open Source", *The Journal of Industrial Economics*, vol. 50, n. 2, June 2002, p. 197-234.

Mailath, G. & Postlewaite A. (1990): "Asymmetric Information Bargaining Problems with Many Agents", *Review of Economic Studies*, 57, p.351-367.

Raymond E.S. (1996): *The Cathedral & the Bazaar: Musings on Linux and Open Source by an Accidental Revolutionary,* available at http://manybooks.net/titles/raymondericother05cathedralandbazaar.html.

Samuelson, P. (1954): "The pure theory of public expenditure", *Review of Economics and Statistics*, 36, p.387-389.

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