

Preface

The Value of Information in Economics and Decision Theory

A Continuous Framework Linking Risk, Uncertainty, and the Scientific Foundations of Economics

This book presents a new framework for understanding economic decision-making by integrating information theory, risk, and uncertainty into a continuous, optimization-based approach. Drawing on the foundational contributions of Claude Shannon and Ruslan Stratonovich, the manuscript develops the tools needed to distinguish the pure value of acquiring information from its cost in decision problems. This separation allows for a rigorous treatment of the trade-offs between informational investment and the expected outcomes of economic decisions.

The core of the book is a comprehensive theory of the Value of Information (VoI), where uncertainty is modelled as an information-processing constraint that affects the structure of optimal decision-making following Stratonovich (1965, 2020). As observed in foundational work such as Green and Laffont (1979, 1986), economic theory since the 1970s has focused increasingly on misaligned objectives among agents, with applications in principal-agent theory, taxation, insurance, employment contracts, and auction design.

While modern economic theory has largely focused on strategic information asymmetries, with hidden types, incentive compatibility, and mechanism design at the core (e.g., Holmström (1979) or Myerson (1981)), ear-

lier approaches treated information as a continuous signal, modelled analogously to physical systems under uncertainty. Foundational works (e.g., Marschak and Radner (1972) or Hurwicz (1960)) explored decision-making under informational constraints using tools from statistical decision theory and control theory, rather than strategic elicitation.

This tradition was deeply influenced by developments in information theory and stochastic processes, particularly the work of Stratonovich (1963, 1967), whose mathematical formulation of optimal filtering under noise provided a general theory for decision-making in dynamic systems with uncertain, degraded signals. His treatment of information flow, quantified through [Shannon (1948)] entropy and processed via Bayesian updating, laid a conceptual foundation for treating economic behaviour as arising from constrained information processing under imperfect signals.

This broader perspective, also evident in Arrow (1971) and Theil (1967), envisioned economics as an information-processing discipline, where uncertainty arises not from strategic concealment but from structural limits on signal acquisition and processing, a view closer to statistical physics than to traditional game theory. This book seeks to revive and extend that perspective, bridging the conceptual gap between information theory as developed in the classical framework of Cover and Thomas (2006) and the modern treatment of Polyanskiy and Wu (2025), and contemporary economic modelling. We show that, based on the VoI framework, a wide range of settings in game theory, finance, statistics, and beyond can be analysed, and that the same framework is bound, through convex duality Rockafellar (1974, 1970), even to the mathematics of optimal transport.

This book's VoI approach is also closely related to Sims (2003, 2006), Rational Inattention framework, which models agents as bounded information processors. Rather than assuming perfect information or strategic concealment, rational inattention introduces explicit limits on attention and information flow, grounded in information theory. Similarly, the VoI framework treats uncertainty as a constraint on processing and using information, offering a natural way to model decision-making under realistic cognitive and informational limits.

A central theme of the book is the parallel between an information budget and a risk capital constraint. This connection enables the application of VoI theory to financial portfolio problems, uncertainty quantification, and robust optimization under limited information capacity. The frame-

work introduced here not only fills a conceptual gap in classical economic modelling, but also extends the scope of decision theory to data-intensive domains such as machine learning and artificial intelligence.

When utility functions are incorporated, the VoI formulation yields convex risk measures in the sense of Artzner *et al.* (1999) that are extended coherent in the sense of Rockafellar (2007), and can be extended to entropic optimal transport problems. In these settings, information flows are viewed as constrained transport of probability mass, offering a new lens on problems in transport economics, statistical inference, and algorithm design.

A standout feature of this manuscript is its seamless integration of continuous uncertainty into the core optimization paradigm that defines neo-classical economic analysis, as introduced in the first three chapters. This rigorous yet accessible approach makes the book an ideal resource for graduate students, researchers, and professionals in economics, decision theory, and data science. It offers a compelling and unified framework for understanding information, uncertainty, and rational choice, bridging foundational theory with modern applications.

S. Behringer

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PART 1
First Part

Optional Text