Chapter 6

Price Competition between Platforms: The Case of eBay vs. Yahoo! Auctions*

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Abstract

We investigate the equilibrium market structure on competing online auction sites such as those of eBay or Yahoo! Building on the model of Ellison *et al.* (2004), we take full account of the complexity of network effects on such platforms. We extend the model by looking at the implication of exogenous and endogenous buyer and seller charges making use of contingent tariffs. This extension brings in line the theory with the empirical findings of Brown and Morgan (2009) that eBay sellers enjoy higher prices and more favorable buyer–seller ratios than Yahoo! sellers. We also investigate welfare effects and look at the viability of duopoly with size differentials, at large markets, and at implications for policy.

Keywords: Platform competition, two-sided markets, auctions, equilibrium coexistence, eBay.

JEL Classification Codes: L11, L13, D43, D44.

1. Introduction

Virtual market platforms such as auctions often reveal very different price strategies despite the fact that such intermediaries offer homogenous

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products. Competition between eBay and Yahoo! auctions are a case in point with Yahoo! auctions having substantially lower fees and commissions than eBay both in the US and in Japan. Despite these similarities, markets were eventually dominated by eBay in the US and by Yahoo! auctions in Japan (see Yin, 2004).

One explanation for this observation is the presence of network externalities. Intermediation between heterogenous agents such as bargaining buyers and sellers generates direct externalities (from agents of their own type) and indirect network externalities (from agents of the other type). This complex interaction of network externalities often remains unmodeled; an exception being the work "Competing Auctions" of Ellison, Fudenberg, and Möbius (EFM, 2004).[1] The analysis in EFM shows that stable equilibria in such duopoly markets exist and that they may be asymmetric.

EFM provide a simple model of static network competition between two competing auctions, i.e., *two-sided markets* involving buyers and sellers of a homogenous good. The network externalities on such auction markets are diametrically opposed, i.e., a buyer benefits if there are fewer other buyers in his/her auction (negative own-side effects or congestion externalities) and if there are more sellers (positive cross-side effects or indirect network effects). Inversely, a seller benefits from having more buyers and fewer sellers in his/her auction platform. The magnitudes of these effects can be calculated and compared as auctions are well-understood bargaining processes in the economics literature.

EFM have pointed out that when aggregating and netting these network effects, a "scale effect" dominates, i.e., the fact that larger auctions are more efficient than smaller ones implies that losses from own-side effects are more than outweighed by gains from cross-side effects. Still there is a countervailing "market impact effect", i.e., the fact that a single buyer or seller when switching auction platforms will have an impact on the outcome of the auctions by changing the buyer–seller ratio and thus the expected auction price for the good. In conjuction, these two effects allow for the co-existence of small and large auction platforms in equilibrium, i.e., a situation where neither buyers nor sellers on any of the two auctions prefers to switch platforms or abstain from the auction.

[1]Another exception is Nocke *et al.* (2007).

In their Section 7, EFM also look at the consequences of the two auction sites choosing an endogenous *common charge* that is assumed not to violate participation constraints and upset equilibrium allocations. This choice of modeling is partly motivated by the 1975 price-fixing allegations for Sotheby's and Christie's and partly because more general pricing strategies would lead to additional (ad-hoc) assumptions about selection rules in order to tackle the resulting *equilibrium multiplicity*. Whence the consequences of *asymmetry* for optimal platform pricing (that is generically observed in practice absent price-fixing arrangements) were not pursued.[2]

The aim of this chapter is to explain why undifferentiated platforms may charge *different* charges in equilibrium and potentially earn positive profits and what comparative statics these charges will have. We first identify conditions for an equilibrium in the agents' (buyers and sellers) allocation between the two platforms, for *exogenous* entry fees. Intuitively, if platform 2, for example, charges buyers a higher entry fee than platform 1, then the equilibrium allocation should involve a higher ratio of buyers–sellers in platform 1 than in platform 2. This is because platform 1 should become less attractive to buyers (to compensate for its lower entry fee) by including more buyers and less sellers than in platform 2. Similar logic applies for sellers.

The implications of the EFM framework for online trading platform competition between the most important players in the online auction platform markets *eBay* and *Yahoo! auctions* have been investigated in Brown and Morgan (2009).[3] In their extension of the EFM model, they also look at exogenous vertical platform differentiation (as in Shaked and Sutton (1982)) that could result from differing fixed entry fees.

One of their findings (see Proposition 4 in their paper), is that given eBay is the dominant platform and provides an exogenous vertical differentiation advantage to sellers, firstly more buyers are attracted to a given Yahoo! auction than an eBay auction, and secondly prices for the traded goods are higher on Yahoo! auctions than on eBay. The authors note

[2]The objective to also investigate asymmetric price with elastic demands on competing two-sided platforms (albeit not in an auction setting) can be traced back to the foundational work of Rochet and Tirole (2003).

[3]Alternative applications include the competition of exchanges, see Cantillon and Yin (2011).

that both predictions are *exactly contradicted* by their evidence from field experiments. As an alternative, they offer a dynamic disequilibrium model with boundedly rational players that will eventually lead to “tipping”.

In this chapter, we are offering a more parsimonious extension of EFM that is in accordance with equilibrium coexistence taking differences in seller charges into account. This extension is empirically warranted as eBay has almost always been the more expensive platform for sellers in practice, charging listing fees *and* commissions. However, treating pricing/vertical differentiation as exogenous is clearly not fully satisfactory in the context of competing platforms either.

We therefore move on to investigate the effects of *endogenous* seller charges (e.g., entry fees) on the equilibrium market structure. These equilibria may be asymmetric in the proportion of buyers and sellers that join each platform, and in the sellers’ access fees. We are able to show that introducing seller charges and thus vertical differentiation is sufficient to reconcile the empirical findings in Brown and Morgan (2009) with their use of the EFM model independently whether such differentiation is the result of an endogenous competitive outcome or an exogenous advantage. It is thus robust and supported by the empirical findings.

In analogy with the common agency literature, it seems straightforward that platforms as principals should be able to make their actions (the payments required from agents) *contingent* on the choice of the agent, i.e., the observable allocation decision of buyers and sellers. This is strongly related to the idea of *insulating fees* on two-sided platforms (put forward in Weyl (2010) for monopoly and White and Weyl (2012) for competition) where platform pricing may be made conditional on this eventual allocation mitigating the coordination problems.

A major advantage of such contingent tariffs over the employment of equilibrium refinements is that they are flexible enough to accommodate the potentially *asymmetric* market shares of buyers and sellers on each platform and endogenously reflect these asymmetries. While we are aware of the fact that such tariffs are only sufficient to bring about the observed changes in charges as eBay gains market dominance in many empirical markets (see Yin, 2004), we think that the investigation of price menus is warranted in its own right and may open new perspectives for platform operators. For example, it would be technically feasible for eBay to make seller charges contingent on the number of competing products and/or the number of buyers who decide to monitor a given auction on their watchlist.

From a policy perspective, a satisfactory analysis of *asymmetric* competition in such two-sided markets is clearly of vital importance as it reflects the most pressing cases of policy concern. The fact that effective competition and anti-trust policy in standard markets generically do not carry over to two-sided markets with strong indirect network effects is well established (see e.g., Wright, 2004) and hence satisfactory and robust theoretical models have an impact on regulation policy.

The economic literature on two-sided markets in particular on platform competition has been blossoming recently. Auctions sites as special kinds of platforms with a clear and well-understood game structure are thus amenable to this analysis. Based on the pioneering work of Caillaud and Jullien (2003) and Armstrong (2006), one focus has been to try to deal with the inherent *multiplicity of equilibria.* These may arise for example from coordination failures, congestion externalities as in EFM, the possibility of multi-homing, i.e., joining multiple platforms, and/or simple price setting processes where, even in monopoly, equilibrium prices need not be unique despite the fact that equilibrium allocations of agents are (and indeed would determine unique equilibrium prices).

In order to tackle the issue, early research has resorted to *belief restrictions* such as "bad-expectation beliefs" (originating in Caillaud and Jullien (2003), and refined in Armstrong and Wright (2007)) such that agents are assumed to join one particular platform unless it is a dominant strategy for them not to do so. The consequences of such "responsive" as compared to "passive" expectations and hybrid forms of monopoly and duopoly pricing have been investigated recently in Hagiu and Halaburda (2013).

Viewing the problem of platform competition as a (multi) principal-agent problem (albeit with important externalities) with platforms trying to induce players to take certain actions using take-it-or-leave-it offers, the issue has parallels with the common agency literature originating in Bernheim and Whinston (1986). In an incomplete information setting, Halaburda and Yehezkel (2013) are employing these parallels using a *sequential* game under "*ex-post* asymmetric information", i.e., the same informational structure as EFM. Argenziano (2008) and Jullien and Pavan (2012) are investigating these issues within a global-game framework.

A paper that is close to our motivation to explain the coexistence of asymmetric outcomes in two-sided markets is Ambrus and Argenziano (2009). While their paper also starts with heterogenous consumers, the informational structure differs strongly as agents know their types when choosing the platform already. Also due to their assumption that agents are atomistic, the reasons for finding equilibrium coexistence are fundamentally different.

2. The Model

We model the duopolistic platform competition departing from a simple two-stage game presented in EFM (2004).

The timing of the game is as follows: In the first stage, B risk-neutral buyers ($B \in \mathcal{N}_0$) with unit demand and S risk-neutral sellers ($S \in \mathcal{N}_0$) with a single unit of the good to sell and no reservation value simultaneously decide whether to attend platform 1 or platform 2. In the second stage, buyers learn their valuations that are uniformly i.i.d. distributed on the unit interval and the object is auctioned off. We model this bargaining as a uniform price (multiobject if $S > 1$) auction on each platform. By the revenue equivalence theorem, this choice of the bargaining process is quite general. Each buyer only demands one homogeneous good. In order to guarantee strictly positive prices, we make the "non-triviality assumption" that

$$B > S + 1, \tag{1}$$

for both being positive integers. Sellers have their expected utility given by the expected price on their chosen platform. A buyer's utility on a platform with B buyers and S sellers is given by his/her expected net utility conditional on winning the good i.e.,

$$u_B = E\left\{v - v^{S+1,B} \middle| v \geq v^{S,B}\right\} \Pr\left\{v \geq v^{S,B}\right\}, \tag{2}$$

where $v^{k,n}$ gives the k highest order statistic of a draw of n values and thus in this auction format, the uniform price is simply the $S+1$ highest of the buyers valuations $v^{S+1,B}$ (i.e., the highest losing bid). This is the typical mathematical convention as long as we deal with a discrete model.

Larger markets are more efficient than smaller ones as they come closer to the *ex-post* efficient outcome to allocate a good to a buyer if and only if his/her valuation is high. The *ex-post* efficient outcome implies that the buyers with the S highest values obtain the good, so that the expectation of the maximum total *ex-ante* surplus (welfare) is

$$B \Pr\left\{v \geq v^{S,B}\right\} E\left\{v \middle| v \geq v^{S,B}\right\} = SE\left\{v \middle| v \geq v^{S,B}\right\}$$
$$= SE\left\{v \middle| v > v^{S+1,B}\right\} = S \int_0^1 \left(\int_x^1 v f(v | v > x) dv \right) f^{S+1,B}(x) dx, \tag{3}$$

where $f^{S+1,B}$ is the density function of $v^{S+1,B}$, the $S+1$ highest order statistic of a draw of B values under the uniform distribution.

Lemma 1 (EFM) *Under the uniform distribution, total welfare on one platform can be written as the sum of buyer and seller utilities*

$$w(B, S) = S\left(1 - \frac{1}{2}\frac{1+S}{B+1}\right) = S\left(\frac{B-S}{B+1}\right) + B\left(\frac{S(1+S)}{2B(B+1)}\right).$$

Proof. See EFM (2004). □

The result is intuitive: The total value of a sale is the expected value of v *given* $v > p$, where p is the equilibrium price of the product within the platform. Under the uniform distribution, this is $1 - \frac{1}{2}\frac{1+S}{B+1} = p + \frac{1-p}{2}$. Clearly, the second term is the value for one buyer with the remaining p (as calculated above) going to the seller. To obtain total welfare, we multiply with the number of sales.

Note that

$$\frac{\partial w(1, \frac{S}{B} = \bar{x} < 1)}{\partial B} = \frac{1}{2}\bar{x}\frac{(2-\bar{x})(B+2)B+1}{(B+1)^2} > 0, \tag{4}$$

showing that for constant shares of sellers to buyers, larger markets are more efficient than smaller ones. The efficiency deficit makes it more difficult for small markets to survive but the sequential structure of the game allows for equilibria with two active platforms whenever the impact of switching of buyer and/or seller on his/her expected surplus more than outweighs the efficiency advantage.

The game is solved by backward induction and the solution concept is Subgame Perfect Nash equilibrium (SPNE). The transaction of the good in stage two yields *ex-ante* utility in stage one for a seller of

$$u_S(B, S) = p = \frac{B - S}{B + 1}, \tag{5}$$

and for a potential buyer of

$$u_B(B, S) = \frac{1 - p}{2}\frac{S}{B} = \frac{S(1 + S)}{2B(1 + B)}. \tag{6}$$

Note that holding S/B (the relative advantages of buyers and sellers) constant, sellers prefer larger, *more liquid* markets (where the expected equilibrium price is higher) and buyers prefer small, less efficient markets as

$$\frac{\partial u_S(1, \frac{S}{B} = \bar{x} < 1)}{\partial B} = \frac{\partial p(1, \frac{S}{B} = \bar{x} < 1)}{\partial B} > 0, \tag{7}$$

and

$$\frac{\partial u_B(1, \frac{S}{B} = \bar{x} < 1)}{\partial B} < 0. \tag{8}$$

Extending the setting of EFM, we assume that platforms can charge buyers and/or sellers some fee for participating but do not allow for price-discrimination, i.e., all buyers and sellers are assumed to face the same fee. Remember that consumers choose homogenous platforms *ex-ante*, i.e., before knowing their types. This *ex-ante* homogeneity of agents makes price discrimination using a “divide and conquer” type strategy by (heterogenous) platforms targeting specific subsets of heterogenous consumers (with possibly negative prices) as in Jullien (2011) a less obvious possibility and, on the other hand, voids the need for sophisticated belief refinements to counter the resulting multiplicity of equilibria.

As buyers and sellers simultaneously decide which platform to join in stage one, we can set up the relevant constraints that determine the set of *all* possible SPNE of the game subject to the qualification that the integer constraint holds. Otherwise, we will speak of a *quasi-equilibrium*.

This restriction is investigated in detail in Anderson *et al.* (2010). The constraints to keep buyers in place in stage one given buyer charge difference $p_{2B} - p_{1B} \equiv \Delta_B \geq 0$ are

$$u_B(B_1, S_1) \geq u_B(B_2 + 1, S_2) - \Delta_B, \tag{B1}$$

and

$$u_B(B_2, S_2) - \Delta_B \geq u_B(B_1 + 1, S_1). \tag{B2}$$

In words: A buyer on platform 1 needs to have an expected utility from the auction stage correcting for charges paid to the platform owner such that a change to the other platform and the implied effect on the equilibrium outcome of the auction there deters him from doing so.

To keep sellers in place in stage one given seller charge difference $\Delta_S \geq 0$, we need

$$u_S(B_1, S_1) \geq u_S(B_2, S_2 + 1) - \Delta_S, \tag{S1}$$

and

$$u_S(B_2, S_2) - \Delta_S \geq u_B(B_1, S_1 + 1), \tag{S2}$$

to hold. The motivation for the constraints is analogous. Clearly, these constraints matter only for interior equilibria.

2.1. *Exogenous buyer charges*

We now look explicitly at the form of the constraints and thus at the set of possible SPNE with some exogenous charge differences $\Delta_B > 0$ to (winning) buyers in auction two. Note that this does not imply that charges are made only by one of the platforms but only that it is the difference between such charges that influence location incentives.

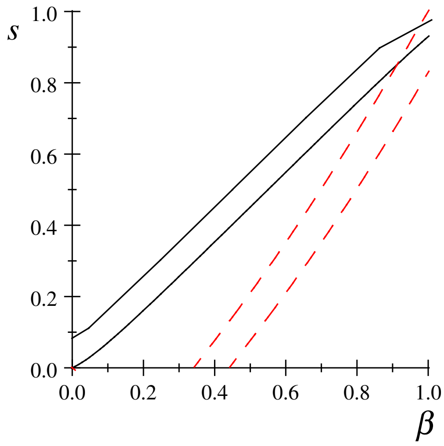
Denoting s as the *share* of sellers on platform 1 and β as the *share* of buyers at platform 1, the buyer constraint (B1) becomes

$$\frac{sS(1+sS)}{2\beta B(1+\beta B)} \geq \frac{(1-s)S(1+(1-s)S)}{2((1-\beta)B+1)(1+(1-\beta)B+1)} - \Delta_B,$$

and (B2) is

$$\frac{(1-s)S(1+(1-s)S)}{2(1-\beta)B(1+(1-\beta)B)} - \Delta_B \geq \frac{sS(1+sS)}{2(\beta B+1)(1+\beta B+1)}.$$

A numerical example (with $B = 10$, $S = 5$) illustrates how the buyer constraints change. The two buyer constraints with $\Delta_B = 0$ (*solid lines*) and $\Delta_B = 0.3$ (*dashed lines*) are:



where the share of sellers on platform 1 (s) is on the ordinate and the share of buyers on platform 1 (β) is on the abscissa.

The interpretation of this finding is as follows: The lower *solid line* is the (B1) constraint gives the condition that buyers stay on platform 1 if the fraction of sellers s is large enough or, alternatively if β is low enough. The higher solid line is the (B2) constraint gives the condition under which buyers stay on platform 2, i.e., if s is small (and thus $(1-s)$ the fraction of seller on his/her own platform is large enough). Between the two curves is the candidate set of SPNE (we still need to check if the seller constraints hold).

Now with a charge of $\Delta_B > 0$ to buyers on platform 2 both the (B1) and the (B2) constraint shift downwards to the *dashed lines*, i.e., the set of SPNE allows for equilibria with a lower share of sellers on platform 1 for a given share of buyers. The (B2) constraint also shifts downwards. i.e., buyers move from platform 2 at higher levels of s already (and thus for a lower fraction of seller $(1 - s)$ on his/her own platform) than before given the new charge.

2.2. *Exogenous seller charges*

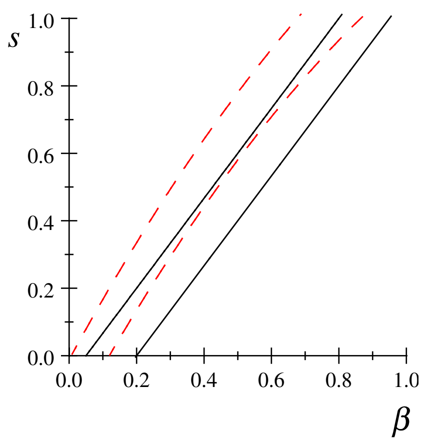
We now introduce an exogenous charge difference Δ_S for sellers of platform 2. Seller constraints are

$$\frac{\beta B - sS}{\beta B + 1} \geq \frac{(1-\beta)B - ((1-s)S + 1)}{(1-\beta)B + 1} - \Delta_S, \tag{S1}$$

and

$$\frac{(1-\beta)B - (1-s)S}{(1-\beta)B + 1} - \Delta_S \geq \frac{\beta B - (sS + 1)}{\beta B + 1}. \tag{S2}$$

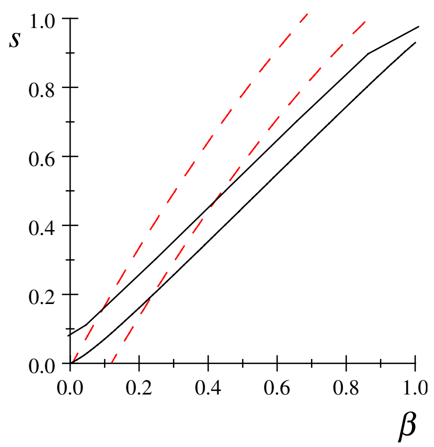
With $\Delta_S = 0$ (*solid lines*) and $\Delta_S = 0.3$ (*dashed lines*), we find the picture with the seller constraints becomes:



The interpretation of this finding is as follows: For the upper *solid line* is the linear (S1) constraint, a seller stays on platform 1 if s is not too high for a given share of β, otherwise he will go to platform 2. For the lower solid line, the linear (S2) constraint, a seller stays at platform 2 if s is high (i.e., his/her own seller share $1 - s$ is low), otherwise he will go to platform 1. Between the two curves is the candidate set of SPNE (we need to check if the buyer constraint holds simultaneously).

Now that there is a charge of $\Delta_S > 0$ to the sellers on platform 2, the (S1) constraint is no longer linear and shifts upwards to the upper *dashed line*: Sellers stay on platform 1 even if s is much higher than before for given β. Similarly, the (S2) constraint is no longer linear and also shifts upwards to the lower dashed line: Sellers will move from platform 2 even if s is much higher (hence their own seller share $1 - s$ much lower) than before.

The numerical example with $\Delta_S = 0.3$ (*dashed lines*) and the original buyer constraints for $\Delta_B = 0$ *(solid lines)* yields both seller and buyer constraints as:



Only $\beta = 0.2, s = 0.2$ is a viable equilibrium here and the previous candidate $\beta = 0.4, s = 0.4$ is no longer viable.

The result reveals that charging sellers on platform 2 allows for higher s tolerance for given β on platform 1. Also, equally sized platforms are no longer viable. As sellers like larger, more liquid platforms where the uncertainty about the resulting final price is lower, we find that a positive and exogenous relative seller charge difference of platform 2 can only be an equilibrium if platform 2 also has the larger share of sellers.

3. Endogenous Seller Price Competition

We next look at duopoly competition where platforms may charge sellers for using the platform only. We focus on seller charges mostly because of the major application of this chapter, i.e., the competition between eBay and Yahoo! auctions where eBay explicitly forbids seller-fee-shifting. The assumption also holds for many other examples of platform competition and hence we consider it the more interesting variant. Again, we abstract from the possibility of price-discrimination.[4]

In order to investigate the issue, we extend setup and timing of the game: In the *first stage*, platforms $j = 1, 2$ simultaneously choose seller charges that may be *contingent* on realized allocations of buyers and sellers, i.e., a *function* that specifies charges as a function of buyer and seller allocations, $p^{S,j}, p^{S,-j}\colon (\beta, B, s, S) \to \mathbb{R}^+$.

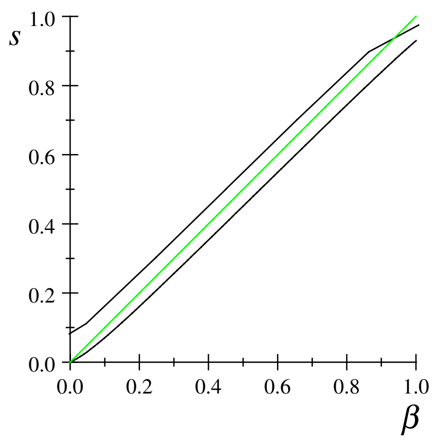
In the *second stage*, buyers and sellers simultaneously decide which platform to join, learn their valuations that are uniformly distributed on the unit interval and the object is auctioned off. Payoffs to buyers, sellers, and platforms are realized (this was described as a separate stage above but can be collapsed into a single stage as we have shown for the EFM model above).

We make use of a very weak equilibrium refinement for the second stage: Instead of implementing belief/expectation restrictions or a new solution concept we define:

Definition 1 *An "efficient equilibrium" for the EFM model is the welfare maximizing equilibrium taken from the set of stable candidate equilibria.*

It is then possible to show that the efficient equilibrium will be *unique*. Intuitively as can be seen from the example above, the buyer switching constraints absent any buyer charges are "tight" around the diagonal, 45° line, i.e., there is no room for additional candidate equilibria neither above or below, nor to the right or to the left of the diagonal:

[4]Note that with endogenous "insulating fees" for which we allow here, it could be a profitable strategy to target initial sellers with a fee for a small number of buyers that is small (or even negative) but is then increasing (and positive) for a higher number of buyers. This would convince all sellers to join which in turn implies that all buyers will, in line with the idea of a "divide and conquer" strategy. This strategy also seems empirically plausible (e.g., *early adopters* with technical products) but again we abstract from such price discrimination due to a lack of buyer heterogeneity and because we believe that such sophisticated temporal strategies should most profitably be investigated in a truly dynamic framework (see Cabral, 2012).



The buyer switching constraint thus forces all candidate equilibria to be *on* the diagonal. Now from what EFM call the "scale effect", we know that independently of the interplay of the four network effects (direct, indirect on both platforms), the aggregate effect is such that welfare is always maximized at the corner i.e., for a monopoly situation and that this monotony implies a unique efficient candidate equilibrium.[5] It is again obvious from this finding that any relevant policy implication from this model needs to take into account charges and thus the distributive issues that would result from a monopoly situation.

In order to discuss price formation in the above platform game, we now introduce platforms' *pricing strategies* to the first stage of the model that have the spirit of the *insulating fees* used in Weyl (2010) and White and Weyl (2012) (i.e., a mapping or menu from buyer and seller allocations into charges) to select their "*target allocations*" which mitigates the coordination problems. Lee (2013) also investigates contingent transfers in a one-sided setting. In the continuation equilibrium, the buyers and sellers then choose the platforms contingent on these price mappings. Platform profits are simply

$$\Pi_j = S_j p^{S,j} \quad \text{for } j = 1, 2, \tag{9}$$

where the charge $p^{\cdot}$ can be interpreted as a markup net of seller unit costs.

[5]For more details, see Behringer (2012).

Definition 2 *A "seller pricing strategy" is a map that specifies the charge to sellers as a function of the buyer and seller platform choices:*

$$p^{S,1}(B_1, B_2, S_1, S_2) \quad \textit{and} \quad p^{S,2}(B_2, B_1, S_2, S_1),$$

or

$$\Delta^S(\beta, B, s, S).$$

This allows charges to be used as a non-cooperative tool to induce coordination on a desired outcome. The solution concept for the first stage is then *pure strategy Nash equilibrium* so that

$$p^{S,j*} = p^{S,j}(B_j^*, B_{-j}^*, S_j^*, S_{-j}^*) \quad \text{for } j = 1, 2 \text{ when } -j = 2, 1,$$

denotes the SPNE of the two-stage game where platforms maximize profits holding the other platforms price map constant and anticipate correctly that buyers and sellers will play the unique *efficient* equilibrium in stage two.

In most analyses of platform competition, it is assumed that *participation constraints* of the buyers are always met. For example, in a Hotelling model where the fixed benefit always outweighs possibly high charges by platforms. An explicit treatment of participation constraints often obscures results substantially (e.g., Lee, 2013, fn.11) or renders solutions implausible. On the contrary, we are able to investigate participation constraints and profit conditions (feasibility) in detail below.

Furthermore, demand also should depend on charges: If platform j charges a high enough seller charge, then it should lose all its sellers. In order to meet these intuitive requirements, we will assume the following assumption:

Assumption 1. Agents do react to price changes of platforms in a way that their incentive (switching) constraints remain binding. Hence, a bounded and non-trivial demand elasticity can be calculated.

Proposition 1 *The equilibrium seller charge difference (of the price maps) resulting under competition and A are given by*

$$\Delta^{S*R} = -3\frac{B_1 - B_2 - S_1 + S_2 + B_1S_2 - B_2S_1}{(B_2 + 1)(B_1 + 1)},$$

if charges are rigid (R) and by

$$\Delta^{S*F} = -3\frac{2B_1 - B_2 - S_1 + S_2 + B_1 S_2 - B_2 S_1 + 1}{(B_2+1)(B_1+1)},$$

if charges are fully flexible (F).

Proof. See Appendix. □

The equilibrium target charges have the expected comparative statics as:

$$\frac{\partial p^{S_1,1*}}{\partial S_1} = -\frac{1}{B_1+1} < 0, \quad \frac{\partial p^{S_1,1*}}{\partial B_1} = \frac{S_1+2}{(B_1+1)^2} > 0$$

$$\frac{\partial p^{S_1,1*}}{\partial S_2} = \frac{2}{B_2+1} > 0, \quad \frac{\partial p^{S_1,1*}}{\partial B_2} = -\frac{2S_2+1}{(B_2+1)^2} < 0.$$

The comparative statics with flexible charges are identical except

$$\frac{\partial p^{S_1,1*}}{\partial B_2} = -\frac{2S_2+3}{(B_2+1)^2} < 0,$$

which differs in magnitude but not in sign. We focus on the rigid charge case in what follows.

3.1. *Participation constraints*

Note that (27) as calculated in the Appendix can be written as

$$p^{S_1,1*} = \frac{(2\beta B+2)(1-s)S - ((1-\beta)B+1)sS + (\beta B - 2(1-\beta)B - 1)}{((1-\beta)B+1)(\beta B+1)}. \tag{10}$$

The participation constraints of a *seller on platform 1* is thus

$$U^{S1} = \frac{B_1 - S_1}{B_1+1} - p^{S_1,1*} = \frac{2B_2 - 2S_2 + B_1 B_2 - 2B_1 S_2 + 1}{(B_2+1)(B_1+1)} \geq 0, \tag{11}$$

which does not depend on S_1 anymore. The comparative statics reveal:

$$\frac{\partial U^{S1}}{\partial S_2} = -\frac{2}{B_2+1} < 0, \quad \frac{\partial U^{S1}}{\partial B_1} = -\frac{1}{(B_1+1)^2} < 0,$$
$$\frac{\partial U^{S1}}{\partial B_2} = \frac{2S_2+1}{(B_2+1)^2} > 0,$$

so that taking price competition into account, the seller utility now decreases in the number of sellers on the other platform, decreases in the number of buyers on its own, but increases in the number of buyers on the other platform.

Expressing the constraint in terms of shares we find,

$$\frac{\partial U^{S1}}{\partial S} = -\frac{2(1-s)}{B-B\beta+1} < 0, \quad \frac{\partial U^{S1}}{\partial s} = \frac{2S}{B-B\beta+1} > 0,$$

which clearly follows from the fact that U^{S1} does not depend on S_1 anymore. The remaining statistics are

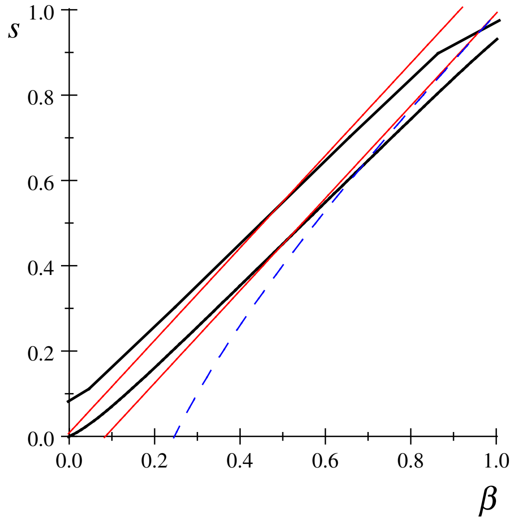
$$\frac{\partial U^{S1}}{\partial B} = \frac{\begin{array}{c}(\beta(1-\beta)(2\beta+2S\beta(1-s)-1))B^2+(4S\beta(1-\beta)(1-s))B \\ +2S(1-\beta)(1-s)+(1-2\beta)\end{array}}{(B(1-\beta)+1)^2(B\beta+1)^2},$$

and

$$\frac{\partial U^{S1}}{\partial \beta} = B\frac{\begin{array}{c}(2\beta(1-\beta-S\beta(1-s))-1)B^2+(4S\beta(s-1)-2)B \\ +2(S(s-1)-1)\end{array}}{(B(1-\beta)+1)^2(B\beta+1)^2},$$

which are increasing and decreasing in large markets respectively.

If we introduce the seller participation constraint (11) into the example above ($B = 10$, $S = 5$), we get the following picture with new switching constraints for endogenous charges (solid line) and the participation constraint for sellers on platform 1 (dashed line):



The example motivates the more general proof.

Lemma 2 *The participation constraint for sellers will be satisfied in any candidate equilibrium.*

Proof. See Appendix. □

3.2. *Platform profits*

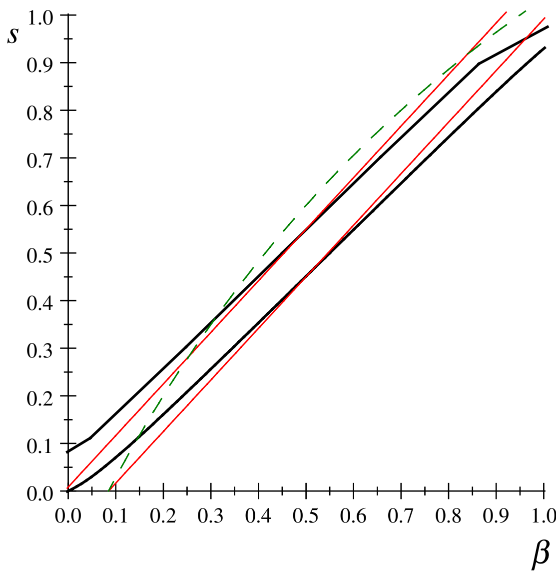
Profits for platform 1 are given by

$$\Pi^1 = p^{sS,1}sS = \frac{\begin{array}{c}\big[(2\beta B + 2)(1-s)S - ((1-\beta)B + 1)sS \\ + (\beta B - 2(1-\beta)B - 1)\big]\end{array}}{((1-\beta)B + 1)(\beta B + 1)} sS,$$

which is non-negative and thus feasible if the allocation in interior and platform equilibrium charges are non-negative. As seller charges on platform 1 (10) are decreasing in s, non-negative profits imply

$$s \leq \frac{2S - 2B + 3B\beta + 2BS\beta - 1}{3S + BS + BS\beta}. \tag{12}$$

For the example above ($B = 10$, $S = 5$), we find the zero-profit line (dashed line) as:



Note that despite the fact that with endogenous charges, from the consumer perspective all equilibria along the diagonal are now feasible, a non-negative-profit constraint on firms implies that extreme outcomes are still not possible and cannot be in the set of targeted allocations. The example generalizes to showing that extreme outcomes will be feasible:

Lemma 3 *The participation constraint for platforms cannot be satisfied for extreme candidate equilibria.*

Proof. Extreme candidate equilibria have β, $s \rightarrow 0$ (or $\beta, s \rightarrow 1$) respectively. We can solve the non-negative profit constraint (12) for β

at equality as

$$\beta = \frac{1}{3B + 2BS}(2B - 2S + 1) > 0,$$

i.e., strictly positive by non-triviality. Hence, extreme candidate equilibria cannot satisfy the feasibility constraints for duopoly. □

The above analysis finds that equilibria of this game may have non-Bertrand outcomes (despite homogeneity of the technology of the platforms and the product of the transaction) where pricing differences between the two platforms can prevail in subgame perfect equilibrium.

Making use of insulating fees and focusing on efficient equilibria allows to tackle the inherent multiplicity of equilibria in this two-sided market setting. As in Section 7 EFM, we focus on endogenous seller charges only but contrary to their analysis, we are able to investigate *asymmetric* insulating seller fees and are able to explicitly analyze asymmetric equilibrium outcomes. We also check for participation constraints and profit conditions without making additional assumptions about selection rules or employ belief restrictions.

4. Implications for Brown and Morgan (2009)

For very large platforms, it is quite intuitive that we cannot observe any charge differentials as the friction of the EFM model and hence the "market impact" effect vanishes. This finding is shared with Proposition 3 in Brown and Morgan (2009) who show that with exogenous vertical differentiation (i.e., a charge difference $\Delta_S > 0$ in our case) equilibrium coexistence in very large markets is impossible.

As the paper by Brown and Morgan is motivated by a detailed empirical and theoretical investigation of the competition between eBay and Yahoo! auctions, we phrase the following findings in this duopoly setting in order to make our findings more easily commensurable. A version of their Proposition 4 holds that in addition:

Proposition 2 *In any quasi-equilibrium in which the sites coexist and eBay (here* 1*) enjoys an* exogenous *vertical differentiation advantage for sellers (here* $\Delta_S > 0$*) and a more than 50% market share, relatively more sellers* *are attracted to a given eBay auction than an Yahoo! auction for sufficiently many buyers.*

Proof. The seller constraint (S2) from (S2) can be transformed into

$$s_{S2} \geq \frac{1}{S(B+2)}(S - 2B + \Delta_S + B\Delta_S - 1) + B\beta\frac{S + B\Delta_S - B\Delta_S\beta + 3}{S(B+2)}. \tag{13}$$

Also given participation constraints hold the maximal advantage for sellers is bounded by

$$\Delta_S < \max_{\beta,s}\left\{\frac{(1-\beta)B - (1-s)S}{(1-\beta)B + 1}\right\}. \tag{14}$$

Now we show that if $\Delta_S > 0$ and $\beta > 1/2$, then $s > \beta$ for sufficiently many buyers.

Note that s_{S2} is strictly concave in β given that $\Delta_S > 0$. The difference between s_{S2} and the 45° line (where $\beta = s$) is

$$d \equiv s_{S2} - \beta = \frac{1}{S(B+2)}(S - 2B + \Delta + B\Delta - 1) + \beta\frac{3B - 2S + B^2\Delta - B^2\Delta\beta}{S(B+2)}, \tag{15}$$

and still strictly concave in β. Thus, the difference d attains a maximum at

$$\beta_{\max} = \frac{3B - 2S + B^2\Delta_S}{2B^2\Delta_S} > 0, \tag{16}$$

at $\beta = 0$, the difference is

$$d = \frac{1}{S(B+2)}(S - 2B + \Delta + B\Delta - 1),$$

which is strictly negative given (14). As derivatives are smooth, there exists a unique intermediate value of β such that $d = 0$. This value can be found as

$$\beta_k = \frac{S - B(2 - \Delta_S) + \Delta_S - 1}{2S - B(B\Delta_S + 3)}. \tag{17}$$

Note that

$$\frac{\partial \beta_k}{\partial \Delta_S} = -\frac{(3B - 2S + B(B+2)(2B - S))}{(\Delta B^2 + 3B - 2S)^2}, \tag{18}$$

which given non-triviality $B > S + 1$ is negative. Hence, the difference is falling in Δ_S. Then there is a critical level of seller advantage such that the critical level of the intersection of s_{s2} and the 45° line is exactly at $\beta = 1/2$. This level is

$$\Delta_k = \frac{B + 2}{2B + B^2 + 2}, \tag{19}$$

and is *falling* in B. With sufficiently many buyers for any $\Delta_S > \Delta_k (\to 0)$, we have given $\beta > 1/2$ that $s > \beta$, i.e., the seller switching constraint ($S2$) can only be satisfied strictly above the 45° line. Thus, platform 1 faces $s > \beta$ and so a seller–buyer ratio of

$$\frac{sS}{\beta B} > \frac{S}{B}, \tag{20}$$

and by adding up

$$\frac{(1 - s)S}{(1 - \beta)B} < \frac{S}{B} < \frac{sS}{\beta B}. \tag{21}$$

□

The original Proposition 4 in Brown and Morgan holds that with an *exogenous* vertical differentiation advantage for sellers and a more than 50% market share, relatively more buyers are attracted to a Yahoo! than an eBay auction (platform 1) which contradicts their data. Once charges are endogenized using Proposition 6, we note that the theoretical implication is *exactly reversed* as sellers on eBay will actually face relatively more favorable buyer seller ratios as they will have to pay the higher seller charge and thus face an endogenous vertical differentiation disadvantage (i.e., $\Delta_S < 0$).

It turns out that endogenizing charges to sellers is also sufficient to reverse the result in Brown and Morgan (2009) about the relative *transaction prices* on both platforms and hence bring the model in line with their data in this respect too. We can show that:

Proposition 3 *In any quasi-equilibrium in which the sites coexist and eBay (here* 1*) enjoys an* endogenous *vertical differentiation* disadvantage for sellers *(here* $\Delta^{S*F} < 0$*) and a more than 50% market share, for sufficiently many buyers, the transaction price on eBay is higher than that on Yahoo! auctions.*

Proof. See Appendix. □

Hence, we find that with endogenous seller charges the transaction prices on eBay will be larger than those of Yahoo! auctions, in line with the data findings in Brown and Morgan (2009).

Alternatively, we can also use an exogenous vertical differentiation advantage for *buyers* to similarly show that:

Proposition 4 *In any quasi-equilibrium in which the sites coexist and eBay (here* 1*) enjoys an* exogenous *vertical differentiation* advantage for buyers *(here* $\Delta_B > 0$*) and a more than 50% market share, relatively more buyers are attracted to a given eBay auction than an Yahoo! auction for sufficiently many buyers and the transaction price on eBay is higher than on Yahoo! auctions.*

Proof. See Appendix. □

We have thus shown two alternatives by which the empirical results reported in Brown and Morgan (2009) can be brought in line with the theory. The first implies that the liquidity effects of a large market will dominate the effect of the endogenous seller charges on large platforms leading to a higher expected transaction price to the detriment of its buyers. Alternatively, one may argue that if eBay has an exogenous vertical differentiation advantage for buyers in addition to being the dominant platform in a liquid market, this is also sufficient to explain the more favorable buyers–seller ratio for its buyers and for it to have larger transaction prices than at Yahoo! auctions.

Note that endogenous seller charges satisfy

$$\lim_{B\to\infty}(\Delta^{S*F}) = \lim_{B\to\infty}\left(3\frac{(B-S)(1-2\beta)}{(B\beta+1)(B(1-\beta)+1)}\right) = 0. \tag{22}$$

The intuition for this limit result is straightforward: The possibility that the switching of either buyer or seller has a tangible impact on expectations decreases as the number of buyers and sellers increases so that in the limit as markets get very large, all friction disappears from the model and we get a Bertrand type outcome with regard to the charge differences and proportional equilibria. This proposition can be easily extended to an unspecified distribution of valuations and is thus robust.

We also have a result for welfare on large platforms: As total welfare of a platform goes out of bounds, if the platform gets very large, we look at total welfare per buyer and seller respectively

$$\frac{w(B,S)}{B} = u_B(B,S) + \bar{x}u_S(B,S) = \bar{x}\left(1-\frac{\bar{x}}{2}\right), \tag{23}$$

and

$$\frac{w(B,S)}{S} = \frac{1}{\bar{x}}u_B(B,S) + u_S(B,S) = 1-\frac{\bar{x}}{2}, \tag{24}$$

where $\bar{x}$ is the limit of the total seller to buyer ratio. By the non-triviality assumption, the per capita welfare contribution of a buyer is thus always lower than that of a seller.

5. Conclusion

Often buyers cannot be charged for participating on a platform as for example on eBay where seller-fee-shifting is not allowed. Alternatively, the final transactions may not be observable as on used-car platforms. In these cases, the strictness of the buyer switching constraints implies that independently of whether or not there are charges to the sellers, the equilibrium market structure of the platform duopoly will imply proportional equilibria. This strongly restricts the set of equilibria of the game in EFM.

The original Proposition 4 in the paper by Brown and Morgan (2009) exactly contradicts their data which finds that: "eBay sellers enjoy higher prices and more favorable buyer–seller ratios than do Yahoo! sellers." Endogenizing the platform's pricing decision for sellers using the motivation for an "insulating equilibrium", we are able to show that theory and practice actually reveal an endogenous vertical disadvantage for sellers on eBay being the dominant platform. This observation exactly reverses their theoretical findings bringing them in line with the data from their field experiments.

A similar finding pertains with respect to the predicted relative transaction prices on both platforms. Once charges to sellers are endogenized, being the dominant platform implies that transaction prices will indeed be larger on eBay, the more liquid platform, again as observed in their field experiments. An alternative theoretical derivation of these results can be observed for an exogenous vertical differentiation advantage for buyers on eBay.

In conclusion, we have been able to explain why undifferentiated platforms may charge different prices in equilibrium. Our extension of the EFM model taking into account optimal platform pricing behavior results in a unique equilibrium target allocation and, as its mirror image, a set of participation-contingent equilibrium seller charges that fits the empirical results of Brown and Morgan (2009) for competition in the online auction market between eBay and Yahoo! auctions.

6. Appendix

Proof of Proposition 1: The game is solved using backward induction. W.l.o.g. we will focus on platform 1 with profits $\Pi_1 = S_1 p^{S_1,1}$. Using A, we can assume that (S1) is strictly binding thus

$$\frac{\beta B - sS}{\beta B + 1} - p^{S_1,1} \geq \frac{(1-\beta)B - ((1-s)S+1)}{(1-\beta)B+1} - p^{S_2+1,2}, \tag{25}$$

can be written as

$$p^{S_1,1} = p^{S_2+1,2} + \frac{B_1 - S_1}{B_1 + 1} + \frac{S_2 - B_2 + 1}{B_2 + 1}. \tag{26}$$

We investigate two cases:

(a) Prices are *rigid* (R) so that platforms cannot change their prices quickly and hence

$$p^{S_2+1,2} = p^{S_2,2}.$$

Substituting into the profit condition, the concave program for the optimal target allocation is

$$\max_{S_1} \Pi_1 = S_1 \left(p^{S_2+1,2} + \frac{B_1 - S_1}{B_1 + 1} + \frac{S_2 - B_2 + 1}{B_2 + 1} - c^{S,1} \right),$$

with the necessary and sufficient first-order conditions yielding best responses

$$S_1^{BR} = \frac{1}{2} (B_1 + 1)\, p^{S_2+1,2} + \frac{1}{2} \frac{2B_1 - B_2 + S_2 + B_1 S_2 + 1}{B_2 + 1},$$

and by symmetry

$$S_2^{BR} = \frac{1}{2} (B_2 + 1)\, p^{S_1,1} + \frac{1}{2} \frac{2B_2 - B_1 + S_1 + B_2 S_1 + 1}{B_1 + 1}.$$

Solving simultaneously the equilibrium target allocation then has

$$S_1^* = \frac{1}{3} (B_1 + 1) \left(p^{S_1,1} + 2p^{S_2+1,2} + \frac{3}{B_2 + 1} \right),$$

and

$$S_2^* = \frac{1}{3} (B_2 + 1) \left(p^{S_2+1,2} + 2p^{S_1,1} + \frac{3}{B_1 + 1} \right).$$

These can be implemented with *conditional* equilibrium charges

$$p^{S_1,1*} = \frac{(2B_1 + 2)S_2 - (B_2 + 1)S_1 + (B_1 - 2B_2 - 1)}{(B_2 + 1)(B_1 + 1)}, \tag{27}$$

and

$$p^{S_2+1,2*} = \frac{(2B_2 + 2)S_1 - (B_1 + 1)S_2 + (B_2 - 2B_1 - 1)}{(B_1 + 1)(B_2 + 1)}, \tag{28}$$

yielding

$$\Delta^{S*FP} = p^{S_2+1,2*} - p^{S_1,1*} = -3\frac{B_1 - B_2 - S_1 + S_2 + B_1 S_2 - B_2 S_1}{(B_2+1)(B_1+1)},$$

with rigid prices, as given above.

(b) If prices are *flexible* (F), we have the Nash targets still as

$$S_1^* = \frac{1}{3}(B_1+1)\left(p^{S_1,1} + 2p^{S_2+1,2} + \frac{3}{B_2+1}\right),$$

and again by symmetry

$$S_2^* = \frac{1}{3}(B_2+1)\left(p^{S_2+1,2} + 2p^{S_1,1} + \frac{3}{B_1+1}\right).$$

These can be implemented by

$$p^{S_1,1} = -2p^{S_2+1,2} + \frac{3B_2S_1 + 3S_1 - 3B_1 - 3}{B_1 + B_2 + B_1B_2 + 1},$$

but now prices adjust so that

$$p^{S_2+1,2} = -2p^{S_1,1} + \frac{3B_1(S_2+1) + 3(S_2+1) - 3B_2 - 3}{B_1 + B_2 + B_1B_2 + 1},$$

and equilibrium charges are

$$p^{S_1,1*} = \frac{3B_1 - 2B_2 - S_1 + 2S_2 + 2B_1S_2 - B_2S_1 + 1}{B_1 + B_2 + B_1B_2 + 1},$$

and

$$p^{S_2+1,2*} = -\frac{3B_1 - B_2 - 2S_1 + S_2 + B_1S_2 - 2B_2S_1 + 2}{B_1 + B_2 + B_1B_2 + 1},$$

which are no longer symmetric. The charge difference for flexible prices is then

$$\Delta^{S*F} = -3\frac{2B_1 - B_2 - S_1 + S_2 + B_1S_2 - B_2S_1 + 1}{(B_2+1)(B_1+1)},$$

as given above. □

Proof of Lemma 2: We show that the participation constraint will remain outside the S2 switching constraint in general. Note that (11) can be written as

$$\frac{2(1-\beta)B - 2(1-s)S + \beta B(1-\beta)B - 2\beta B(1-s)S + 1}{((1-\beta)B+1)(\beta B+1)} \geq 0,$$

which can be solved as

$$s \geq \frac{1}{2S + 2BS\beta}(-2B + 2S + B^2\beta^2 + 2B\beta - B^2\beta + 2BS\beta - 1).$$

The RHS is an increasing and strictly concave function in β as

$$\frac{\partial RHS}{\partial \beta} = \frac{1}{2}\frac{B}{S(B\beta+1)^2}(B^2\beta^2 + 2B\beta + B + 3) > 0,$$

and

$$\frac{\partial^2 RHS}{\partial \beta^2} = -\frac{B^2\,(B+2)}{S\,(B\beta+1)^3} < 0.$$

It thus takes its maximum value at $\beta = 1$ where the constraint becomes

$$s_{PC} \geq \frac{1}{2S + 2BS}(2S + 2BS - 1).$$

The $S2$ constraint is from above

$$\frac{(1-\beta)B - (1-s)S}{(1-\beta)B+1} - \Delta_S \geq \frac{\beta B - (sS+1)}{\beta B + 1}, \tag{29}$$

where, if we replace the equilibrium rigid prices from above and solve for the binding s, we find:

$$s = \frac{1}{4S + 2BS}(2S - B + 3B\beta + 2BS\beta + 1),$$

which is linearly increasing in β. Again, the RHS takes a maximum at $\beta = 1$ for which the condition becomes

$$s_{S2} = \frac{1}{4S + 2BS}(2B + 2S + 2BS + 1).$$

Now if we take the vertical difference to the S1 constraint (29), we find:

$$s_{S2} - s_{PC} = \frac{1}{2}\frac{4B - 2S - 2BS + 2B^2 + 3}{S(B+2)(B+1)},$$

the sign of which depends on the numerator only. It is increasing in B and decreasing in S. As non-triviality equation (1) implies $B > S$ then at most $S = B$ and the numerator becomes $2B+3 > 0$ so the term is always positive. Due to symmetry, the result is sufficient for the participation condition to hold in any equilibrium. □

Proof of Proposition 3: The transaction price difference is

$$\begin{aligned} p_y - p_e &= \frac{(1-\beta)B - (1-s)S}{(1-\beta)B + 1} - \frac{\beta B - sS}{\beta B + 1} \\ &= \frac{B - S - 2B\beta + 2Ss - BS\beta + BSs}{(B(1-\beta)+1)(B\beta+1)}, \end{aligned} \tag{30}$$

which has the same sign as

$$\begin{aligned} \sigma &\equiv B - S - 2B\beta + 2Ss - BS\beta + BSs \\ &= B - S + Ss(B+2) - B\beta(2+S), \end{aligned} \tag{31}$$

which is increasing in s. The (S2) constraint gives

$$\begin{aligned} s_{S2} \geq{} & \frac{1}{S(B+2)}(S - 2B + \Delta_S + B\Delta_S - 1) \\ & + B\beta\frac{S + B\Delta_S - B\Delta_S\beta + 3}{S(B+2)}. \end{aligned} \tag{32}$$

With endogenous and rigid prices, the seller charge differential is

$$\Delta_S^* = 3\frac{(B-S)(1-2\beta)}{(B\beta+1)(B(1-\beta)+1)} < 0, \tag{33}$$

and with many buyers $\Delta_S^* \to 0$. Hence, what remains is the condition

$$s_{S2} \geq \frac{1}{S(B+2)}(S - 2B - 1) + B\beta\frac{S+3}{S(B+2)}. \tag{34}$$

Substituting in the above yields

$$\sigma = -B(1-\beta) - 1 < 0, \tag{35}$$

so this is not sufficient for σ to be positive, we need a higher s. Still we know from the existence of the "scale effect" that the buyer switching constraints are forcing equilibria to be *proportional* so that $\beta = s$ has to hold. The transaction price differential then reduces to

$$p_y - p_e = (B - S)\frac{1 - 2\beta}{(B(1-\beta)+1)(B\beta+1)}, \tag{36}$$

and as $B > S + 1$ and $\beta > 1/2$, we find that this difference is indeed negative. □

Proof of Proposition 4: The transaction price difference is

$$\begin{aligned} p_y - p_e &= \frac{(1-\beta)B - (1-s)S}{(1-\beta)B+1} - \frac{\beta B - sS}{\beta B + 1} \\ &= \frac{B - S - 2B\beta + 2Ss - BS\beta + BSs}{(B(1-\beta)+1)(B\beta+1)}, \end{aligned} \tag{37}$$

which has the same sign as

$$\begin{aligned} \sigma &\equiv B - S - 2B\beta + 2Ss - BS\beta + BSs \\ &= B - S + Ss(B+2) - B\beta(2+S), \end{aligned} \tag{38}$$

which is decreasing in β.

Assuming an exogenous vertical buyer advantage for eBay (1), the (B2) constraint implies

$$\frac{(1-s)S(1+(1-s)S)}{2(1-\beta)B(1+(1-\beta)B)} - \Delta_B \geq \frac{sS(1+sS)}{2(\beta B+1)(1+\beta B+1)}, \tag{39}$$

solving for β implicitly yields

$$2\beta_{B2} \geq 2 + \frac{1}{B} - \frac{\sqrt{\begin{aligned} &B^2\big(2(1+\beta B)(2+\beta B)\Delta_B + sS + sS^2\big) \\ &\quad \times \big(2(1+\beta B)(2+\beta B)\big(\Delta_B + 2S(1-s) \\ &\quad \times (S(1-s)+1)\big) + sS + sS^2\big) \end{aligned}}}{B^2\big(2(1+\beta B)(2+\beta B)\Delta_B + sS + sS^2\big)}. \tag{40}$$

With sufficiently many buyers $2\beta_{B2} \to 2$ so that $\beta > s$. Also

$$\sigma \equiv B - S + Ss(B+2) - B(2+S) = 2Ss - S - BS - B + BSs. \tag{41}$$

The maximum this can take (at $s = 1$) is

$$\sigma = 2S - S - BS - B + BS = -(B - S), \tag{42}$$

which by non-triviality $B > S + 1$ is *always* negative. □

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